Aalto University
School of Science
and Technology

Mining temporal networks

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KDD 2019 tutorial

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tutorial website

https://rozensp.github.io/KDD19-tutorial-temporal

agenda

Part I : introduction and motivation

Part II : models of temporal networks

Part III : group work

Part IV : algorithmic frameworks

Part V : data mining problems

Part VI : future challenges

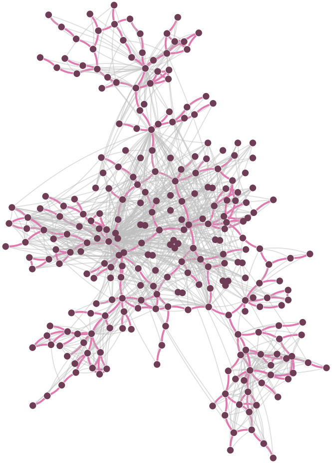
Part VII : group work

part I

introduction and motivation

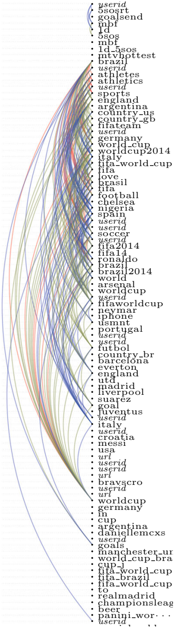
interconnected world

- networks model objects and their relations
- many different network types
  - social
  - informational
  - technological
  - biological
  - …



impact of network science

- online communication networks and social media
- implications in
  - knowledge creation
  - information sharing
  - education
  - democracy
  - society as a whole



research questions in network science

- structure discovery
  - communities, summarization, events, role mining
- study complex dynamic phenomena
  - evolution, information diffusion, opinion formation, structural prediction
- develop novel applications
- design efficient algorithms

traditional view

- networks represented as pure graph-theory objects

  no additional vertex / edge information

- emphasis on static networks

- dynamic settings model structural changes

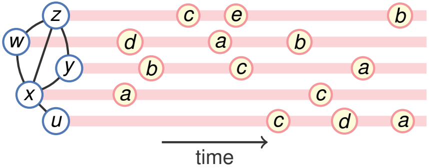
  vertex / edge additions / deletions

temporal networks

- ability to collect and store large volumes of network data
- available data have fine granularity
- lots of additional information associated to vertices/edges
- network topology is relatively stable, while lots of activity and interaction is taking place
- giving rise to new concepts, new problems, and new computational challenges

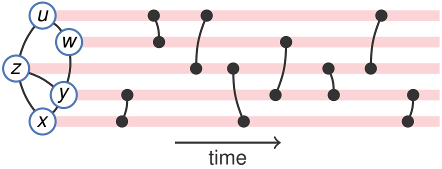
modeling activity in networks

1. network nodes perform actions (e.g., posting messages)

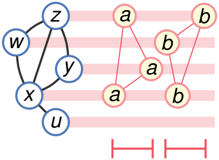


2. network nodes interact with each other

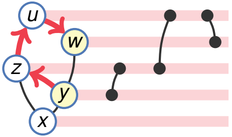
(e.g., a “like”, a repost, or sending a message to each other)



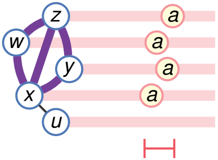
many novel and interesting concepts



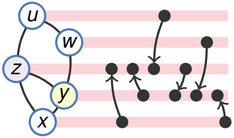
new pattern types



temporal information paths



new types of events



network evolution

temporal networks — objectives

- identify new concepts and new problems
- develop algorithmic solutions
- demonstrate relevance to real-world applications

terminology

- we use term "temporal networks", but terminology is not standardized
- term "X Y" can be encountered in the literature,where

X :	Y :
temporal	networks
dynamic	graphs
(time-)evolving	
time-varying	
time-dependent	
evolutionary	

- some combinations have distinct meaning, but not always

examples of temporal networks

[Holme, 2015]

- human communication networks
  - phone, email, text messages, etc.
- human proximity networks
  - recorded by various sensors and devices
  - bluetooth, wifi, etc.
  - patient-referral networks, i.e. how patients are transferred between wards of a hospital system
  - sexual contact networks
- animal proximity networks
  - obtained via RFID devices
  - lifestock or wildlife

examples of temporal networks — cnt'd

[Holme, 2015]

- bibliographic networks
  - collaboration and citation networks
- economic networks
  - credit card transactions
  - trade networks of countries
  - bitcoin transcations
- travel and transportation networks
  - airline connections, bus transport, bike-sharing systems

examples of temporal networks — cnt'd

[Holme, 2015]

- brain networks
  - temporal correlations of the oxygen levels of brain regions as measured by fMRI scanning
- biological networks
  - genes involved in different interactions that change over time
  - current challenges, as, one cannot measure precisely when two proteins interact with each other, but technology is improving

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part II

models of temporal networks

representation of temporal networks

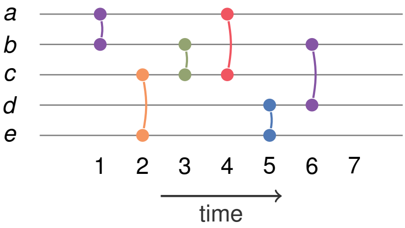
1. sequence of interactions

- a temporal network is represented as $G = (V, E)$
  - with set of nodes V, and
    set of edges $E = \{(u, v, t)\}$, with $u, v \in V$ and $t \in \mathbb{R}$
  - if interactions have duration, then $E = \{(u, v, t, \delta)\}$
- this is a lossless representation — no information is lost
- also known as sequence of contacts, or sequence of (temporal) edges

representation of temporal networks

1. sequence of interactions

- visual representation of a temporal network as a sequence of interactions



representation of temporal networks

2. sequence of static graphs

- sequence $G_1, , \ldots, G_T$

  where $G_t = (V_t, E_t)$, with $t = 1, \ldots, T$

  typically assume that nodes are fixed, i.e., $V_t = V$

  E_t are the edges that occur in time interval t

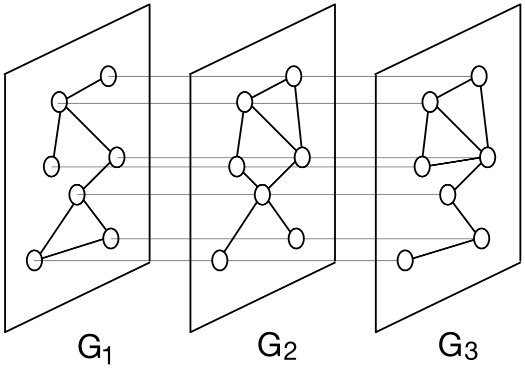
- advantages: static graph analysis methods can be applied

- disadvantages: the representation assumes quantization into time intervals
  - thus, representation depends on quantization parameters, e.g., seconds, minutes, hours, days, etc.
  - coarse resolution may lead to information loss
  - fine resolution may lead to sparse (or even empty) static graphs

representation of temporal networks

2. sequence of static graphs

- visual representation of a temporal network as a sequence of static graphs



representation of temporal networks

3. time series of contacts

– a time-series for each pair of nodes in the network
– equivalent representation with sequence of interactions

4. tensor representation

– tensor representing node $\times$ node $\times$ time information
– can apply powerful tensor-algebra techniques
– a complication is that time is directed, while tensor algebra assumes that indices can be relabeled (breaking the time ordering)

representation of temporal networks

[Casteigts et al., 2012]

5. time-varying graphs defined as $G = (V, E, T, p, \lambda)$, where
   - V : set of nodes
   - $E \subseteq V \times V$: set of edges
   - T : a time domain
   - $p : E \times T \rightarrow \{0, 1\}$: a presence function
   - $\lambda : E \times T \rightarrow \mathbb{R}$: a latency function

- general definition that can be used to model graph datasets in different applications
  - transportation networks, communication networks, social networks

representation of temporal networks

6. stream graphs and link streams [Latapy et al., 2018]

- a formalization for modeling interactions over time
- a stream graph is defined as $G = (T, V, W, E)$, where
  - T : a time domain
  - V : a set of nodes
  - $W \subseteq T \times V$: a set of temporal nodes
  - $E \subseteq T \times V \times V$: a set of links

    s.t., $(t, u, v) \in E$ implies $(t, u) \in W$ and $(t, v) \in W$
- formalization is self-consistent : relations between concepts are preserved
  - e.g., can define clustering coefficient using density
- formalization generalizes usual concepts of graph theory
  - e.g., line graphs, k-cores, cliques, density, centralities

temporal networks vs. dynamic graphs

- dynamic graphs is a standard model typically studied in theoretical computer science
  - e.g., [Henzinger et al., 1999, Thorup, 2000]
- dynamic graphs are represented as a sequence of edge additions and/or edge deletions
- G_0 is the initial graph, and G_i is the graph resulting after the i-th edge addition/deletion operation
- objective: efficient maintenance of graph properties
  - e.g., connectivity, shortest paths, spanners, etc.

temporal networks vs. dynamic graphs

- in dynamic-graph studies, the properties of interest refer to individual graph snapshots G_i, not considering the whole graph evolution
- emphasis on computational efficiency
  - computation time per operation
  - e.g., cost of maintaining a minimum spanning tree per edge additions/deletions
  - or, cost of maintaining a data structure that allows to answer short-path queries
- dynamic graph model captures topological changes, not interactions
  - e.g., dynamic graphs can be used to model friendship additions/deletions in a social network, but not discussions or other interactions

temporal networks vs. dynamic graphs

- dynamic graphs resemble sequence of interactions model
- main difference lies on which graph properties we study
- for dynamic graphs we typically consider properties on graph snapshots
  - i.e., minimum spanning tree on the current snapshot
- for temporal graphs we typically consider properties that span a time interval
  - i.e., a temporal pattern
- disclaimer: in this tutorial we do not consider dynamic graphs
  - however, it is a well-developed area with rich literature

dynamic networks

- in the context of graph generation models, we consider dynamic networks
  - e.g., Barabási-Albert, forest-fire, copying model, etc.
- similar to dynamic graphs, as data are seen as a sequence of node/edge additions (typically no deletions)
- node/edge addition are governed by a probabilistic model, not arbitrary, or worst case, as in algorithmic models
- emphasis again on network topology, i.e., how certain network structures emerge
  - e.g., scale-free distribution, small world, etc.
- disclaimer: in this tutorial we do not consider dynamic networks

graph streams

- setting inspired by data streams
  [Muthukrishnan et al., 2005]
- recall the data-stream model:
  - data are presented as a sequence of data items
    (potentially infinite)
  - assume a small number of passes
    typically constant or just one pass
  - assume small memory compared to data size
    e.g., poly-logarithmic
  - assume fast computation per data item processed
    e.g., constant or poly-logarithmic

graph streams

- a graph stream is a graph dataset in the data-stream model

  e.g., sequence of interactions (temporal network), or sequence of edge additions/deletions (dynamic graph)
- thus, a graph stream is not a representation model, instead it refers to the underlying computational model
- thus, we can study questions of mining temporal networks in the graph-stream model

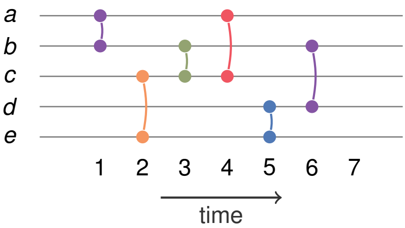
dynamic graph algorithms on streaming model

- well-studied model
- extensive survey [McGregor, 2014]
- different settings considered
  - node/edge additions (incremental)
  - node/edge additions/deletions (fully-dynamic)
  - updating weights/labels is a special case of the fully-dynamic model
  - sliding-window setting: consider only edges from latest interval of fixed length
  - algorithms can be deterministic or randomized

time-respecting paths

- a fundamental concept in analysis of temporal networks
  - used in studies of information propagation, or epidemics spreading
- a time-respecting path is a sequence of temporal edges, such that
  - consecutive edges share a common node, and
  - time stamps of temporal edges are non-decreasing
- intuitively, a piece of information (or disease) can propagate in the network only over time-respecting paths

time-respecting paths — example



$(c, e, 2), (e, d, 5), (d, b, 6)$ is a time-respecting path from e to b

$(c, b, 3), (b, a, 1)$ is not a time-respecting path

static expansion of a temporal network

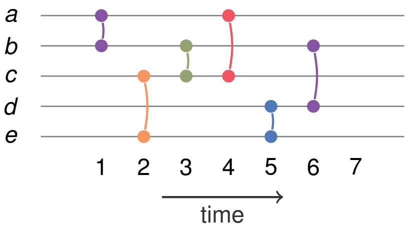
- a transformation of a temporal network to a directed (static) network so that time-respecting paths in the temporal network correspond to directed (static) paths in the directed (static) network

- how to create such a transformation?

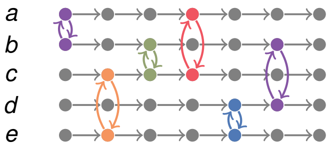
1. create a copy of each node for each time instance
2. create a directed edge from the $(t-1)$-th copy of u to the t-th copy of u, for all nodes u, and all time instances t
3. create directed edges for the temporal edges

static expansion of a temporal network

example



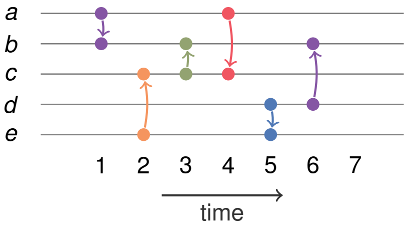
(a) representation of a temporal network



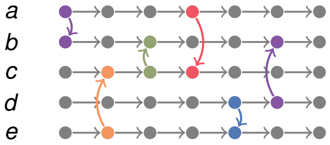
(b) static expansion of temporal network

static expansion of a temporal network

example



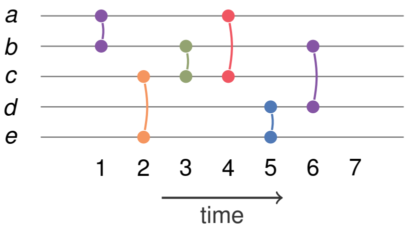
(a) representation of a temporal network



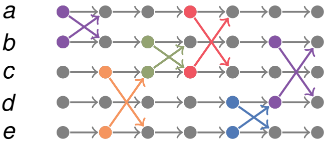
(b) static expansion of temporal network; directed edges

static expansion of a temporal network

example



(a) representation of a temporal network



(b) static expansion of temporal network; delays

reachability, connectivity, and connected components

- defined as in static graphs, but using time-respecting paths
- reachability :
  - used to study infection spreading and information cascades
- connectivity : as in directed (static) graphs is not symmetric
  - distinguish strong and weak connectivity
  - in addition, we can define transitive connectivity:
    a subgraph is transitively connected if time-respecting paths from u to v and v to w imply a time-respecting path from u to w

minimum temporal paths

different notions of minimum temporal paths rely on time-respecting paths

- earliest-arrival path : a path from x to y with earliest arrival time
- latest-departure path : a path from x to y with latest departure time
- fastest path : path from x to y with minimum elapsed time
- shortest path : fastest path from x to y in terms of overall traversal time required on edges

[Wu et al., 2014]

diameter, network efficiency

- diameter : shortest latency of time-respecting paths over connected pairs [Chaintreau et al., 2007]
  - restricted on connected pairs, as real data have many disconnected pairs
- network efficiency : the harmonic mean of latency over all pairs [Tang et al., 2009]
  - discussion : what is the motivation for harmonic mean?

diameter, network efficiency

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  - discussion : what is the motivation for harmonic mean?
  - it combines average latency and reachability ratio

centrality measures

- many centrality measures on static graphs use distances
- closeness centrality : $C_c(u) = \frac{n-1}{\sum_{v \neq u} d(u,v)}$
- betweenness centrality : $C_b(u) = \frac{\sum_{v \neq u \neq w} p_u(v,w)}{\sum_{v \neq u \neq w} p(v,w)}$
- for temporal networks we replace distance with shortest latency time-respecting path
- analogues of Katz centrality and PageRank have also been defined
- discussion : how do these centrality measures on temporal networks compare with their static analogues?

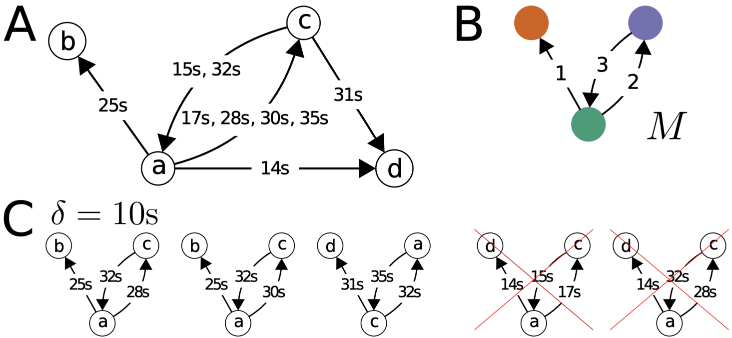
temporal motifs

- temporal motif counting

  [Paranjape et al., 2017, Kovanen et al., 2013]:

  - temporal motif is a small subgraph with temporally ordered edges (and/or interval or delay constraints)

temporal motifs



δ-temporal motif: a sequence of directed temporally ordered edges which appear within a time window δ

[Paranjape et al., 2017]

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part IV

algorithmic frameworks for temporal network analysis

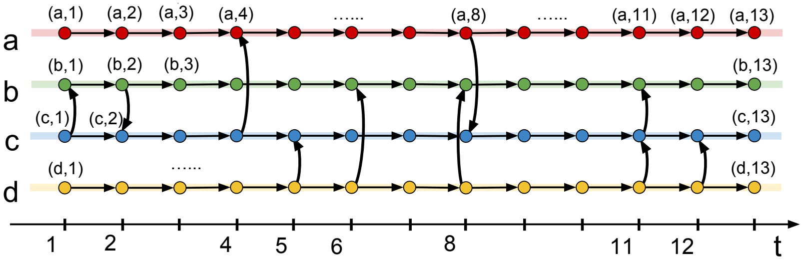
frameworks

adopted traditional frameworks

- static expansion graphs
- dynamic graphs
- time-series
- labeled graphs

static expansion graphs

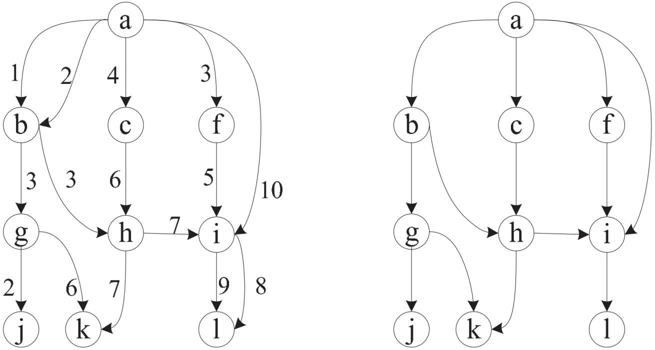
- static graph of time-stamped nodes and time-forwarding edges $G_e = (V_e, E_e)$
- $V_e = \{(v, t) \mid v \in V,\ t \in T\}$, where T is the set of all possible timestamps
- edges E_e : interactions between the temporal nodes V_t



static expansion graphs

- static expansion graph is a directed acyclic graph (DAG)
- standard graph algorithms (BFS, DFS, Dijkstra, Bellman-Ford) can be adopted for finding:
  - fastest temporal paths,
    shortest temporal paths, and
    weighted combinations
  - journeys and walks
- upstream, downstream reachability sets

time-respecting paths



(a) Temporal Graph (b) Static Graph

- some paths in the static graph are not meaningful in the temporal graph
- e.g., $a - b - g - j$ is not time-respecting path
- what is the shortest path from a to ℓ?

[Wu et al., 2014]

minimum temporal paths

different notions of minimum temporal paths rely on time-respecting paths

- earliest-arrival path : a path from x to y with earliest arrival time
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[Wu et al., 2014]

earliest-arrival path

- temporal graph $G = (V, E)$
- source vertex x, starting time t_s
- array T of size $|T|$ to record arrival times to each node
- $T[x] = t_s$ and $T[v] = \infty$, for nodes other than source
- process edges (u, v, t, λ) in temporal order
  - if $t \geq T[u]$ (u is already reached from x)
  - check if current edge creates earliest path from x to v
  - if yes, update $T[v] = \min(T[v], t + \lambda)$

[Wu et al., 2014]

latest-departure path

- temporal graph $G = (V, E)$
- sink vertex x, ending time t_s

- same process as for earliest-arrival path, but
- process edges in reversed temporal order
- add new interaction to the path if it does not violate temporal order

[Wu et al., 2014]

dominating path

- source vertex x and sink v
- define $(a[v], s[v])$, where
  - $a[v]$: time of arrival to v
  - $s[v]$: time of departure from x
- consider another path $(a'[v], s'[v])$
- if ($s'[v] > s[v]$ & $a'[v] \leq a[v]$) or ($s'[v] = s[v]$ & $a'[v] < a[v]$)
  - then path $(a'[v], s'[v])$ dominates path $(a[v], s[v])$
- if there is a path (u_1, u_2) in interval $[t_s, t_e]$ with duration d, which includes $(a[v], s[v])$,
  - then there is path (u_1, u_2) in $[t_s, t_e]$, which is not slower and includes $(a'[v], s'[v])$

[Wu et al., 2014]

fastest path

- source vertex x, list L_v to keep track on path candidates
- define $(a[v], s[v])$, where
  - $a[v]$: time of arrival to v
  - $s[v]$: time of departure from x
- array T to record fastest-path duration for each node
  - $T[x] = 0$ and $T[v] = \infty$, for nodes other than source
- process edges (u, v, t, λ) in temporal order
  - if $u = x$, insert (t, t) into L_x
  - take $(a'[u], s'[u])$ from L_u with the latest arrival time $a'[u]$, so that $a'[u]$ is before t
  - this means we found a new path:

    $a[v] = t + \lambda$ and $s[v] = s'[u]$

    insert this path into L_v
  - remove all dominated paths from L_v
  - update $T[v]$ if this new path is faster than seen so far

[Wu et al., 2014]

shortest path

- similar to algorithm for fastest path
- but keep track on the number of interactions, instead of the duration

[Wu et al., 2014]

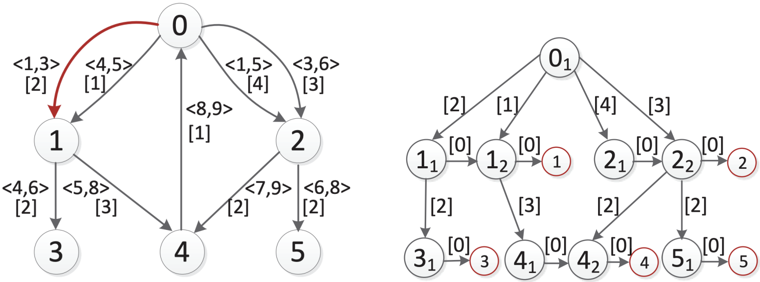
minimum spanning trees

- MST_a : minimum spanning tree with earliest-arrival times

  each temporal path from the root to the node is the earliest arrival path

- MST_w : minimum spanning tree with smallest total weight

  or with the smallest number of hops: directed Steiner tree.

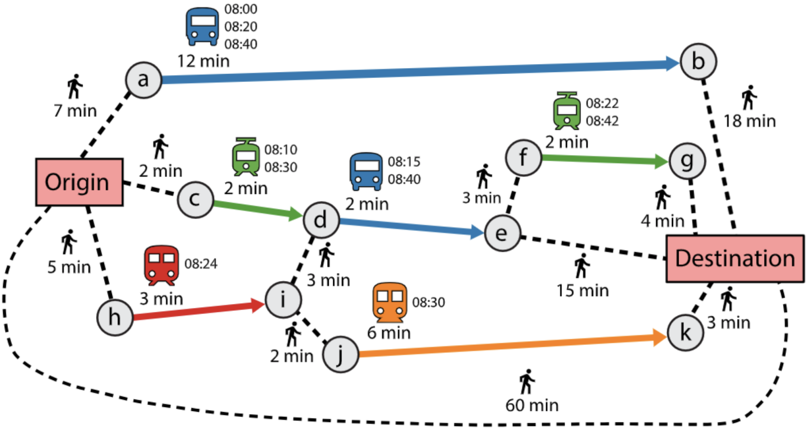


[Huang et al., 2015]

applications of temporal paths

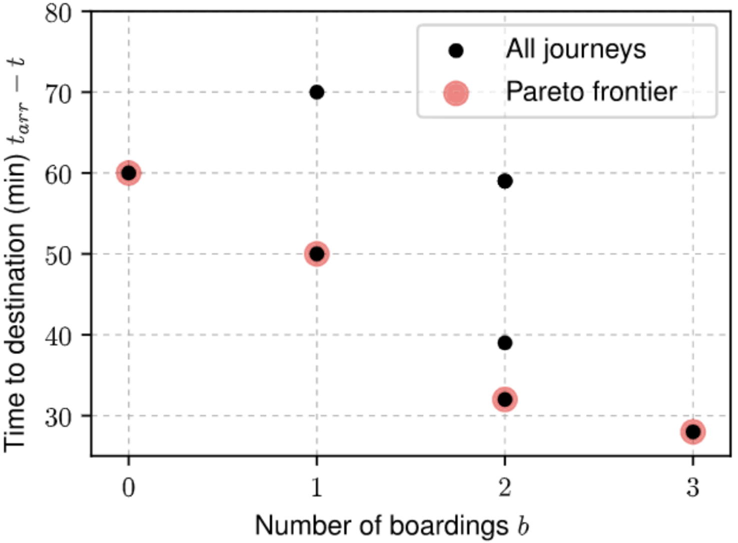
- temporal reachability problems
  - diffusion simulation, centrality measures
- directed spanning or Steiner trees
  - reconstruction of diffusion
- drawback: large size of expansion graph may lead to high computational complexity and large memory consumption
- challenge: scalable algorithms and approximations

applications — transportation temporal networks



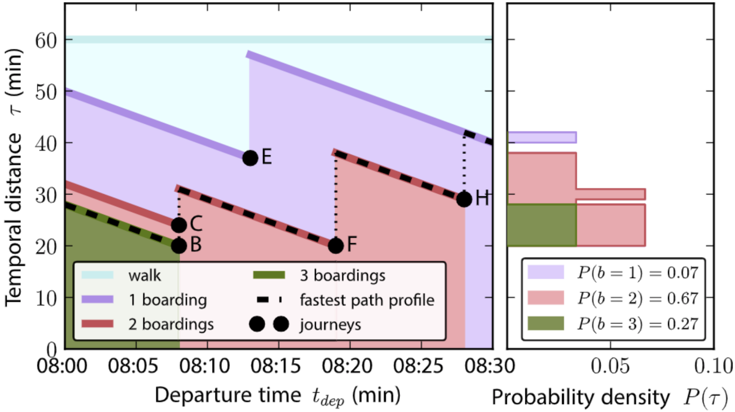
[Kujala et al., 2018]

Pareto-optimal journeys



[Kujala et al., 2018]

Boarding-count-augmented temporal-distance profiles



(a) Boarding-count-augmented fastest-path temporal distance profile

(b) Boarding-count-augmented fastest-path temporal distance distribution

[Kujala et al., 2018]

dynamic graph algorithms on streaming model

- well-studied model
- extensive survey [McGregor, 2014]
- different settings considered
  - node/edge additions (incremental)
  - node/edge additions/deletions (fully-dynamic)
  - updating weights/labels is a special case of the fully-dynamic model
  - sliding-window setting: consider only edges from latest interval of fixed length
  - algorithms can be deterministic or randomized

dynamic graph algorithms on streaming model

[McGregor, 2014]

	Insert-Only	Insert-Delete	Sliding Window (width w)
Connectivity	Deterministic [27]	Randomized [5]	Deterministic [22]
Bipartiteness	Deterministic [27]	Randomized [5]	Deterministic [22]
Cut Sparsifier	Deterministic [2, 8]	Randomized [6, 31]	Randomized [22]
Spectral Sparsifier	Deterministic [8, 46]	Randomized $\tilde{O}(n^{5/3})$ space [7]	Randomized $\tilde{O}(n^{5/3})$ space [22]
$(2t-1)$-Spanners	$O(n^{1+1/t})$ space [11, 23]	Only multiple pass results known [6]	$O(\sqrt{wn^{(1+1/t)}})$ space [22]
Min. Spanning Tree	Exact [27]	$(1+\epsilon)$-approx. [5] Exact in $O(\log n)$ passes [5]	$(1+\epsilon)$-approx. [22]
Unweighted Matching	2-approx. [27] 1.58 lower bound [42]	Only multiple pass results known [3, 4]	$(3+\epsilon)$-approx. [22]
Weighted Matching	4.911-approx. [25]	Only multiple pass results known [3, 4]	9.027-approx. [22]

Table 1: Single-Pass, Semi-Streaming Results: Algorithms use $O(n \operatorname{polylog} n)$ space unless noted otherwise.

sliding-window neighborhood profiles

- temporal network $G = (V, E)$
- stream of edges $E = \langle (u_1, v_1, t_1), (u_2, v_2, t_2), \ldots \rangle$ with $t_1 \leq t_2 \leq \ldots$
- sliding window length w
- snapshot network $G(t, w)$ at time t contains all edges with time-stamps in $(t - w, t]$

problem :

given node u, window length w, and distance r, how many nodes in $G(t, w)$ are within distance r from u at time t?

[Kumar et al., 2015]

proposed online algorithms

1. an exact but memory-inefficient streaming algorithm
2. an approximate memory-efficient streaming algorithm

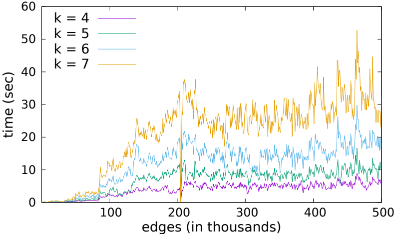
- approximate algorithm uses logic of exact algorithm, combined with hyperloglog sketches
- if number of buckets in the HLL counter is k then the worst case complexity changes to
- update time :

  $\mathcal{O}(rm2^k \log^2 n)$ from $\mathcal{O}(rmn \log n)$
- space complexity :

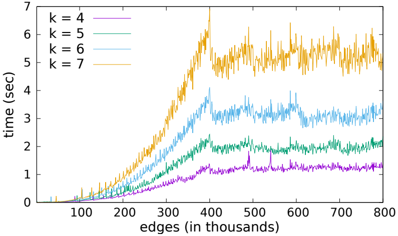
  $\mathcal{O}(rn2^k \log n)$ from $\mathcal{O}(rn^2)$

[Kumar et al., 2015]

empirical evaluation — running time



(c) Higgs



(d) DBLP

contrast (DBLP)

- offline HyperANF : 3.6 sec / sliding window
- proposed approach : 0.003 sec / sliding window

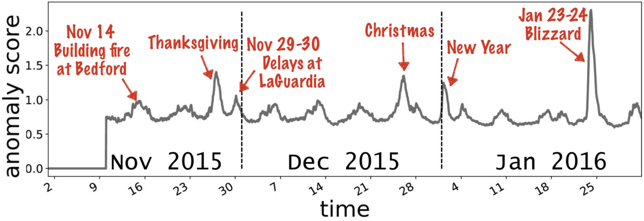
[Kumar et al., 2015]

time-series analysis

- view a temporal network as a multivariate time series
- calculate distance between adjacent snapshots and analyze the resulting time series
- distance: edit distance, node-profile distances, vector-space distance
- applications in change-point detection, anomaly detection, evolutionary pattern mining

event detection in time series

- given a sequence of graphs G_t
- a function to calculate the vertex affinity matrix S, where s_{ij} indicates the influence vertex i has on vertex j
- a set of time points for detected events is
  $\{t \in T \mid d(G_t, G_{t+1}) \geq \delta\}$
  where
  $d(G_t, G_{t+1}) = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} (\sqrt{S_{t,ij}} - \sqrt{S_{t+1,ij}})^2}$



[Eswaran et al., 2018]

time-series analysis

- anomaly detection survey [Ranshous et al., 2015]
- approach does not solve all the problems, as it does not capture the network topology
- possible work-around: use more topology embeddings metrics (larger neighborhoods, influence measures, eigenvectors,...)

labeled graphs

- edges are labeled with occurrence timestamps
- applications of classic graph-theoretical problems
  - coloring, routing, network flow, covering, etc.
- "any property of a graph labeled from a discrete set of labels corresponds to some temporal property if interpreted appropriately" [Michail, 2016]

labeled graphs

- for example, consider a proper edge coloring
  - a coloring of the edges in which no two adjacent edges share a common color
- corresponds to a temporal network where no two adjacent edges share a common time-label
  - i.e., no two adjacent edges ever appear at the same time
- limitation: labels are independent, timestamps are not

theoretical aspects of temporal graphs

- how is the complexity of classic combinatorial optimization problems changes when time is added?
- some old results: the max-flow min-cut theorem holds with unit capacities for time-respecting paths [Berman, 1996]
- a number of recent attempts
  - sliding window vertex cover [Akrida et al., 2018]
  - sliding window graph coloring [Mertzios et al., 2018]
  - maximal matching [Mertzios et al., 2019]

  etc.

theoretical aspects of temporal graphs

- there are many models for abstracting temporal networks
- challenge: which models are most general and most useful?

agenda

Part I : introduction and motivation

Part II : models of temporal networks

Part III : group work

Part IV : algorithmic frameworks

Part V : data mining problems

Part VI : future challenges

Part VII : group work

part V

data mining problems

data mining problems

- community detection
- event detection
- finding important nodes
- epidemics analysis and influence spreading
- network summarization
- ...

community detection

community detection in static graphs

- static graphs: extensive survey [Fortunato and Hric, 2016]
- standard community definitions
  - a community is a set of nodes, which are closer to each other than to the rest of the network
  - a community is a dense network subgraph
- general definition [Coscia et al., 2011]
  - a community in a complex network is a set of entities that share some closely correlated sets of actions with the other entities of the community
- typical problem settings
  - a single community vs. network partition
  - overlapping vs. non-overlapping communities

community detection in static graphs

partition measures

- modularity : the difference between the actual number of edges and the expected
- cut : number of edges between a community and the rest of the graph
- ratio cut : cut normalized by the number of edges of community nodes
- ...

single-community measures

- average degree : $\frac{|E(S)|}{2|S|}$
- density : $\frac{2|E(S)|}{|S|(|S|-1)}$
- conductance : $\frac{cut(S,\bar{S})}{min\{vol(S),vol(\bar{S})\}}$
- ...

community detection in temporal networks

temporal information gives rise to several issues

- temporal localization: concise time interval or intervals, whole time history
- behaviour: single-appearance, recurring, persistent, evolutionary patterns, smoothness
- partition of the topology network vs. partition of the time history
- online vs. offline
- application-specific settings

community detection in temporal networks

- proposed taxonomies
  - [Aynaud et al., 2013]
  - [Aggarwal and Subbian, 2014]
  - [Enugala et al., 2015]
  - [Renaud and Naoki, 2016]
  - [Hartmann et al., 2016]
  - [Rossetti and Cazabet, 2018]
  - [Dakiche et al., 2019]
  - ...

temporal communities : temporal assumptions

prior model, which describes what is the temporal behavior of interesting community structures, e.g.,

- small/large covering intervals of community interactions
- frequent patterns
- persistent patterns

evolutionary patterns : vocabulary

evolutionary patterns of communities in the network

[Dakiche et al., 2019]

- birth
- death
- growth
- contraction
- merge
- split
- continue
- resurgence

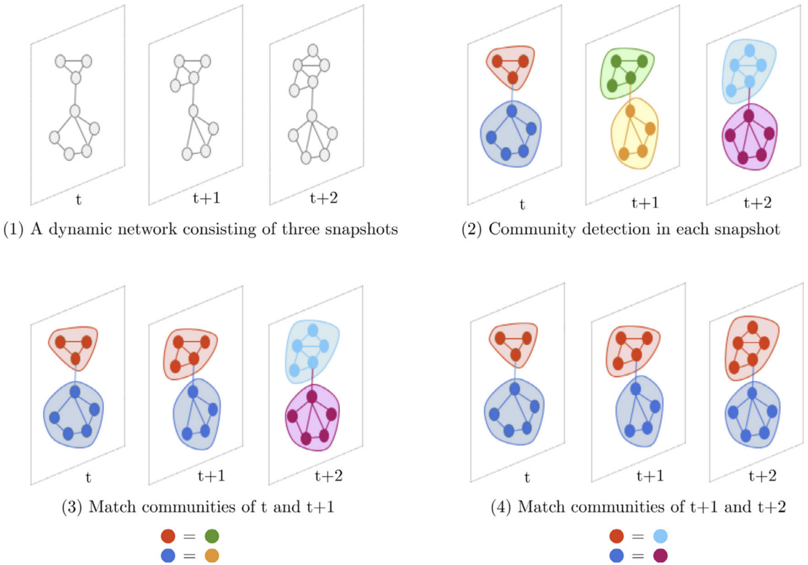
temporal communities: taxonomy

we follow a recent survey on community detection

[Dakiche et al., 2019]

- independent community detection and matching
  - first detect communities at each timestamp
  - then match them across different timestamps

independent community detection and matching



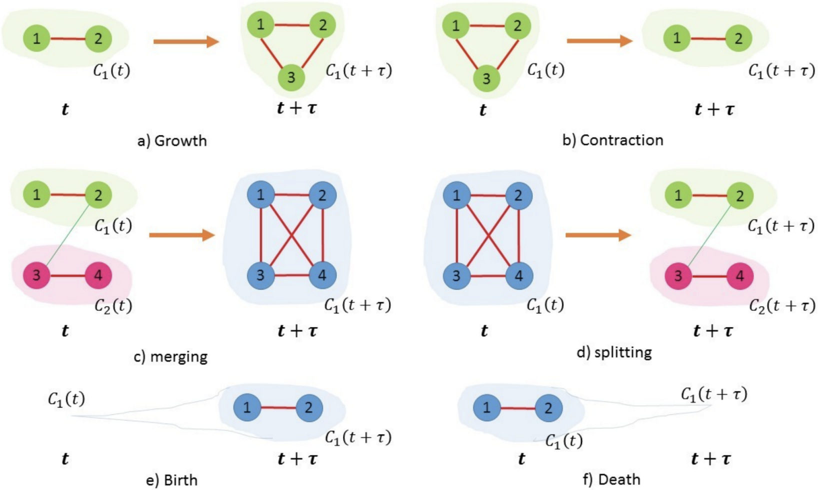
[Dakiche et al., 2019]

independent community detection and matching

Reference	Method	Key elements
Hopcroft et al. (2004)	Agglomerative hierarchical clustering	Similarity between two communities
Asur et al. (2009)	Matching approach	Community membership matrices
Palla et al. (2007)	Joint graphs	Auto-correlation, stationary parameter
Van Nguyen et al. (2012)	Life-cycle model	Jaccard coefficient, minimum community size
Wang et al. (2008)	Life-cycle model (CommTracker framework)	Common core nodes between communities
Chen et al. (2010)	Community core evolution	Maximal cliques, core nodes
Greene et al. (2010)	Step communities	Time step *t*, Jaccard similarity
Takaffoli et al. (2010, 2011)	Event-based model	Community similarity
Bródka et al. (2013)	Group evolution discovery	Inclusion measure
Tajeuna et al. (2016, 2015)	Event-based model	Mutual transition measure
Sun et al. (2015)	Correlation matrix	Communities union
Zhu et al. (2016)	Event-based framework	Community attributes

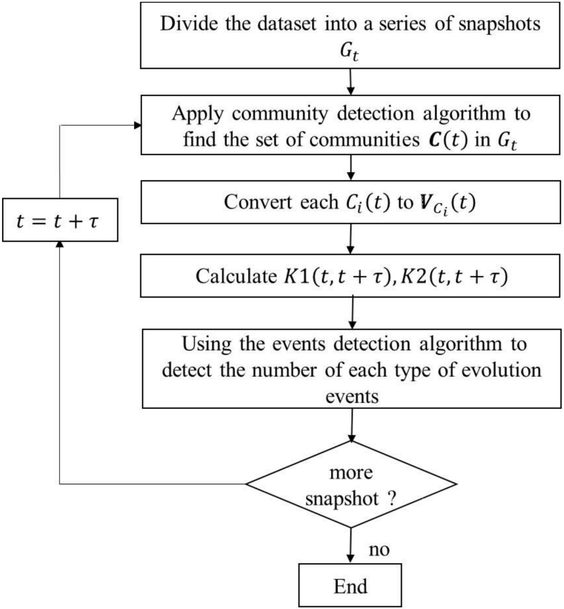
[Dakiche et al., 2019]

typical evolutionary patterns



[Sun et al., 2015]

procedure

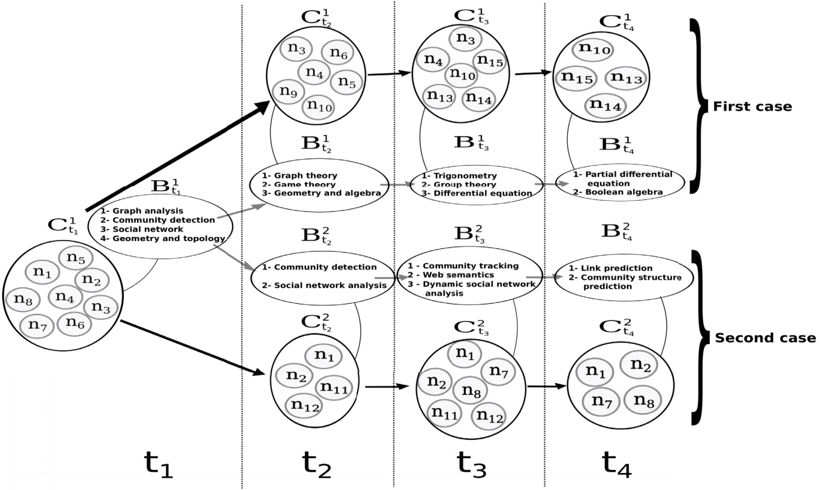

Divide the dataset into a series of snapshots G_t
Apply community detection algorithm to find the set of communities $\boldsymbol{C}(t)$ in G_t
Convert each $C_i(t)$ to $\boldsymbol{V}_{C_i}(t)$
Calculate $K1(t, t+\tau), K2(t, t+\tau)$
Using the events detection algorithm to detect the number of each type of evolution events
more snapshot ?
no
End
$t = t + \tau$

possible issues

- most of approaches investigate the similarity between communities at consecutive time stamps t_i and t_{i+1}
- such an approach may yield a community that does not share any nodes with the initially-observed community
- need to capture temporal relationship

[Tajeuna et al., 2015, Tajeuna et al., 2016]

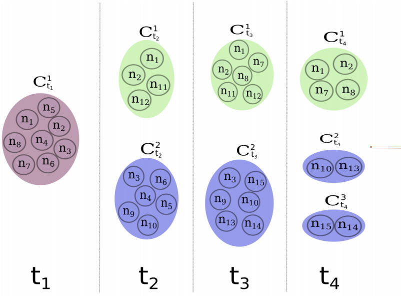
possible issues



[Tajeuna et al., 2015, Tajeuna et al., 2016]

similarity matrix

- $B = A^T \times A$: temporal community similarity matrix (contingency matrix)
- row-normalized : transition matrix, similarity threshold applied
- similarity matrix can be used to track evolution



(a) Community detection at different snapshots.

	t_1	t_2		t_3		t_4		
	C^1_{t1}	C^1_{t2}	C^2_{t2}	C^1_{t3}	C^2_{t3}	C^1_{t4}	C^2_{t4}	C^3_{t4}
n_1	1	1	0	1	0	1	0	0
n_2	1	1	0	1	0	1	0	0
n_3	1	0	1	0	1	0	0	0
n_4	1	0	1	0	0	0	0	0
n_5	1	0	1	0	0	0	0	0
n_6	1	0	1	0	0	0	0	0
n_7	1	0	0	1	0	1	0	0
n_8	1	0	0	1	0	1	0	0
n_9	0	0	1	0	1	0	0	0
n_{10}	0	0	1	0	1	0	1	0
n_{11}	0	1	0	1	0	0	0	0
n_{12}	0	1	0	1	0	0	0	0
n_{13}	0	0	0	0	1	0	1	0
n_{14}	0	0	0	0	1	0	0	1
n_{15}	0	0	0	0	1	0	0	1

(b) Binary membership matrix.

[Tajeuna et al., 2016, Tajeuna et al., 2015]

independent community detection and matching

advantages

- reuses unmodified traditional community detection methods
- possible to use existing similarity measures

disadvantages

- instability of community-detection algorithms

temporal communities: taxonomy

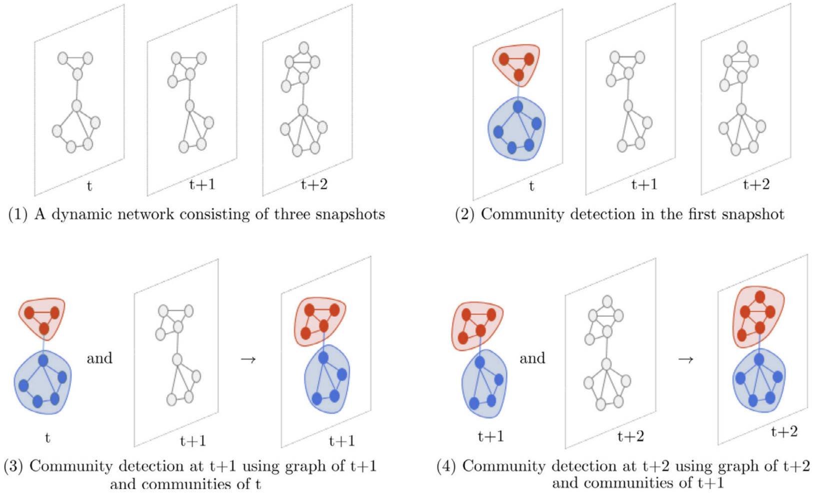
[Dakiche et al., 2019]

- dependent community detection

  detect communities at time t based on

  - network topology at t, and
  - communities at time $t-1$

dependent community detection



[Dakiche et al., 2019]

dependent community detection

Reference	Method	Key elements
He and Chen (2015)	Dynamicity in the Louvain algorithm	Time t
Aynaud and Guillaume (2010b)	Dynamicity in the Louvain algorithm	Time t, communities of $t-1$
Chong and Teow (2013)	Dynamicity in the Louvain algorithm	New nodes
Wang and Fleury (2010)	Dynamicity in the Louvain algorithm	Time t, core nodes of $t-1$
Dinh et al. (2009)	Modularity maximization for dynamic networks	Change in graph snapshot, community structure at time t
Chakrabarti et al. (2006)	Evolutionary clustering	Snapshot quality, history quality
Lin et al. (2009)	Iterative algorithm	Snapshot cost, temporal cost
Yang et al. (2011)	Bayesian inference	Community assignments
Kim and Han (2009)	Nano-communities, quasi-clique-by-clique	Temporal cost, snapshot cost
Chi et al. (2007)	Evolutionary spectral clustering	Temporal cost, snapshot cost
Sun et al. (2007)	Graph encoding	Community structure of last segment
Folino and Pizzuti (2014)	Multiobjective genetic algorithm	Community score, NMI
Rozenshtein et al. (2014)	Iterative method	Time-interval set
Guo et al. (2014)	Attribute information based method	Increments
Gao et al. (2016)	EvoLeaders-based method	Leader nodes

[Dakiche et al., 2019]

Louvain algorithm

- a fast greedy approach based on modularity optimization
- two phases repeated iteratively
  - initially, each node in network is a community
  - then, for each node i, consider its neighbor j and compute the gain of modularity of putting i into the community of j
  - node i is placed into the community with the largest gain, if the gain is positive

[Blondel et al., 2008]

Louvain algorithm

- on the second phase, each community is considered as a super-node
  - the edges between these super-nodes are contracted and re-weighed by the number of edges between them
- the two phases are repeated until there is no improvement in modularity
- the algorithm is extremely fast

[Blondel et al., 2008]

history-dependent approach

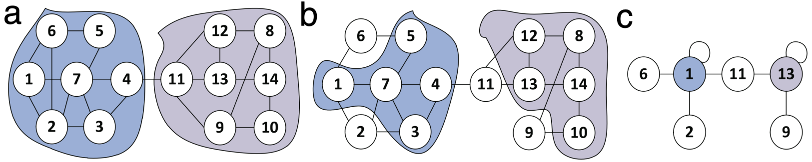
idea

- for two consecutive time steps, there only few edges that affect the community structure
- if the connections of all the nodes in the same community at time step $t - 1$ keep unchanged at time step t, they are still in the same community at time step t
- thus, no need to break that super-node

[He and Chen, 2015]

history-dependent approach

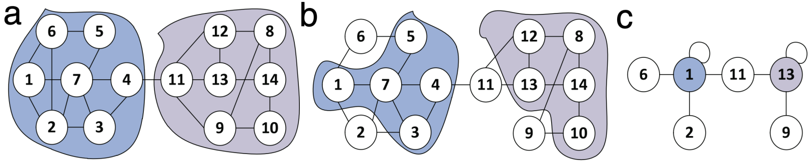
- find all communities in snapshot $t = 1$
- for $t = 2$:
  - if a node's connection change, then remove it from its super-node and add as single node
  - leave all other nodes inside the super-node
  - re-weight the edges



[He and Chen, 2015]

history-dependent approach

- continue Louvain from that point to find communities
- continue in this fashion for $t = 3$ using the communities at $t = 2$, and so on



[He and Chen, 2015]

dependent community detection

advantages

- a solution for the problem of instability
- improved computational complexity

disadvantages

- traditional community detection methods are no longer directly applicable

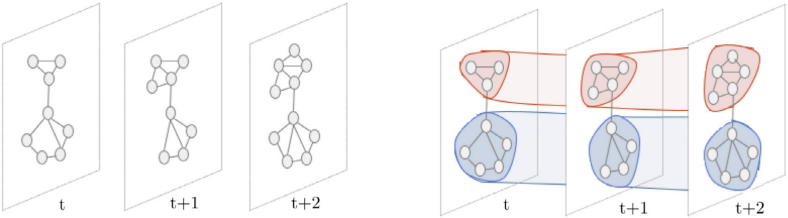
temporal communities: taxonomy

[Dakiche et al., 2019]

simultaneous community detection on all snapshots

- construct a static expansion graph
  - add edges between instances of nodes in different timestamps
- run a standard community detection on the resulting graph

simultaneous community detection on all snapshots



(1) A dynamic network consisting of three snapshots

(2) Community detection on all snapshots

[Dakiche et al., 2019]

simultaneous community detection on all snapshots

Reference	Method	Key elements
Tantipathananandh et al. (2007); Tantipathananandh and Berger-Wolf (2011)	Graph coloring problem, heuristics	Individual cost, group cost, c-cost
Jdidia et al. (2007)	Coupling graph clustering	Group membership
Mucha et al. (2010)	Coupling graph clustering	Modularity measure
Mitra et al. (2012)	Evolution in one graph	Modularity measure
Aynaud and Guillaume (2010a)	Modularity maximization	Average modularity measure

[Dakiche et al., 2019]

simultaneous community detection

- algorithm based on some basic assumptions about individual behavior and group membership

assumptions

- gradual changes : nodes change community affiliation infrequently
- reliable true positive : members of the same community mostly interact with each other
- negligible false positive : members of different communities rarely interact with each other

[Tantipathananandh and Berger-Wolf, 2011]

simultaneous community detection

costs

- switching cost : each node u incurs cost C_{sw} when changing community affiliation
- false negative cost : two nodes incur cost C_{fn} when belong to the same community but do not interact
- false positive cost : two nodes incur cost C_{fp} when belong to different communities but do interact

resulting problem

- find a partition into clusters that minimizes the total cost of switching, false negative, and false positive

[Tantipathananandh and Berger-Wolf, 2011]

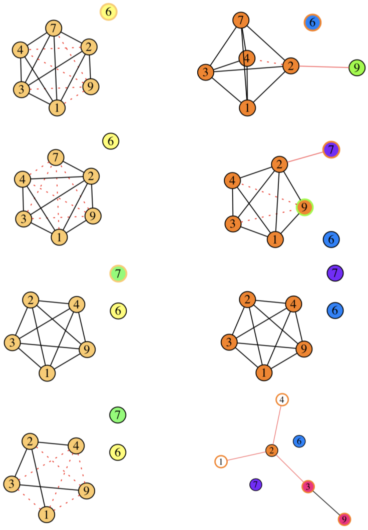
simultaneous community detection

$t = 30$

$t = 31$

$t = 32$

$t = 33$



$(C_{sw}, C_{fn}, C_{fp}) = (5, 1, 5)$ vs. $(C_{sw}, C_{fn}, C_{fp}) = (1, 1, 5)$

[Tantipathananandh and Berger-Wolf, 2011]

simultaneous community detection on all snapshots

advantages

- provides a solution for the problem of instability

disadvantages

- no possibility to track community evolution in a network evolving in real time

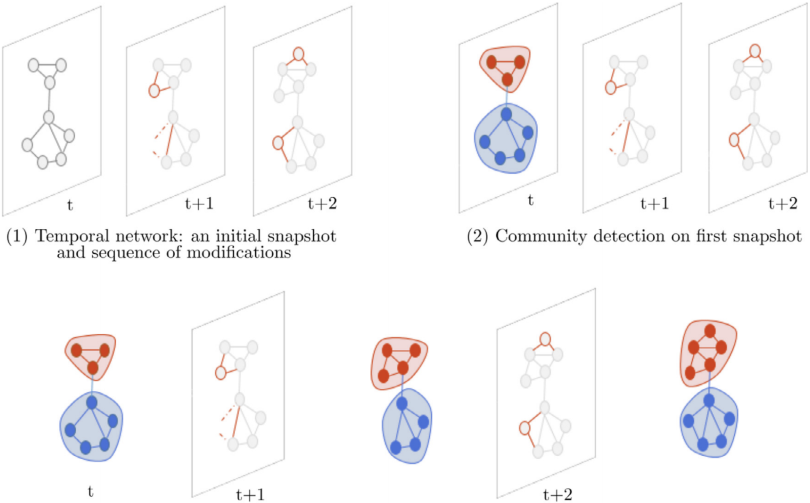
temporal communities: taxonomy

[Dakiche et al., 2019]

dynamic community detection

- update previously discovered communities according to network modifications

dynamic community detection



(1) Temporal network: an initial snapshot and sequence of modifications

(2) Community detection on first snapshot

(3) Update communities of t according to modifications of t+1

(4) Update communities of t+1 according to modifications of t+2

[Dakiche et al., 2019]

dynamic community detection

Reference	Method	Key elements
Li et al. (2012)	Rule-based	Node's number of edges with communities
Shang et al. (2012)	Rule-based	Modularity measure
Cazabet et al. (2010)	iLCD (intrinsic Longitudinal Community Detection)	Path lengths
Nguyen et al. (2011b)	QCA (Quick community adaptation)	Nodes, edges
Nguyen et al. (2011a)	AFOCS	Nodes, edges
Qi et al. (2013)	A probabilistic approach	Trajectory information
Xu et al. (2013)	Rule-based	Edges
Xie et al. (2013a)	Label propagation	Labels of changed nodes
Anita and Bader (2016); Zakrzewska and Bader (2015)	Dynamic seed set expansion	Nodes, fitness score
Lee et al. (2014)	Evolution operations	Bulk updates
Rossetti et al. (2017)	Label propagation	Edges
Bhat and Abulaish (2015)	Novel density-based approach	Core nodes
Cordeiro et al. (2016)	Modularity-based	Nodes, edges
Held and Kruse (2016)	Detection based on high-connected hubs	Nodes, edges
Guo et al. (2016)	Local interaction model	Increments

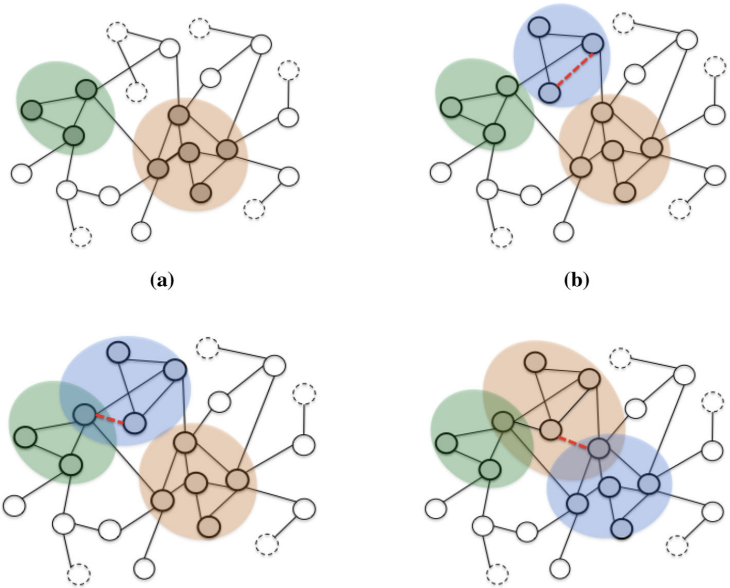
[Dakiche et al., 2019]

TILES

- stream processing
- uses label propagation to diffuse the changes to the node surroundings and adjust neighbors' community memberships
- a node can belong to a community with two different levels of involvement: peripheral membership and core membership
- only core nodes can spread community membership to their neighbors
- edges have a life span threshold, old are removed
- finds overlapping communities, i.e., each node can belong to different communities which can represent the different spheres of the social world of an individual

[Rossetti et al., 2017]

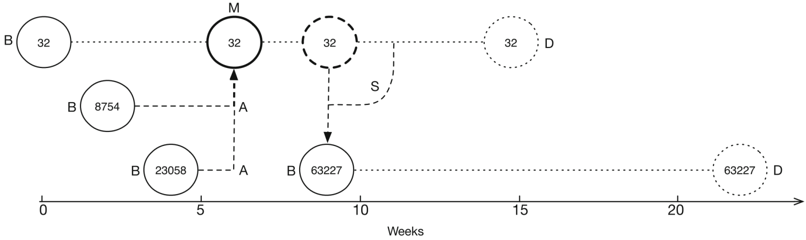
TILES



(a) (b) (c) (d)

TILES

- example of community life cycle extracted from WEIBO
- each community is represented by a circle and identified by an ID
- events of different types
  - (B) birth, (M) merge, (A) absorption, (A) split, (D) death



[Rossetti et al., 2017]

dynamic community detection

advantages

- provides a solution for the problem of instability
- light-weight methods to track communities

disadvantages

- possibility to drift towards invalid communities event detection

event detection

- given a network representing some kind of activity
  - network of social interactions
  - social-media feed
  - transportation network
- an event can be generally defined as an activity with some prominent qualitative or quantitative difference from the background activity
  - bursting news about major natural disasters
  - abnormally high traffic in the city
  - an emerging new discussion topic in social media

applications

- news spread in social media faster than in traditional news media [Sakaki et al., 2010, Dou et al., 2012]
- weather or traffic condition warning systems
- early notification about influential social events
- understanding causal relations, semantics, and dynamics of what is happening

comprehensive survey on event detection in dynamic networks [Ranshous et al., 2015]

temporal event detection

- identify atypical time intervals and/or time instances
- temporal records
  - time sequences (time-ordered records) or
  - time series (equally-spaced in time sequences)
- number of interactions, tweets, reposts, purchases, check-ins, or some other measures in absolute values or aggregated per time unit

temporal event detection

- time series may represent a temporal network
  - topological characteristics of each snapshot
  - distance between two consecutive graph snapshots

temporal event detection: standard approaches

abnormality score

- the likelihood that an interval contains an event can be estimated by comparing an abnormality score on the interval

  [Heins and Stern, 2014]

predictive models

- learn a predictive model and find intervals and time points whose behavior differ from the predicted one

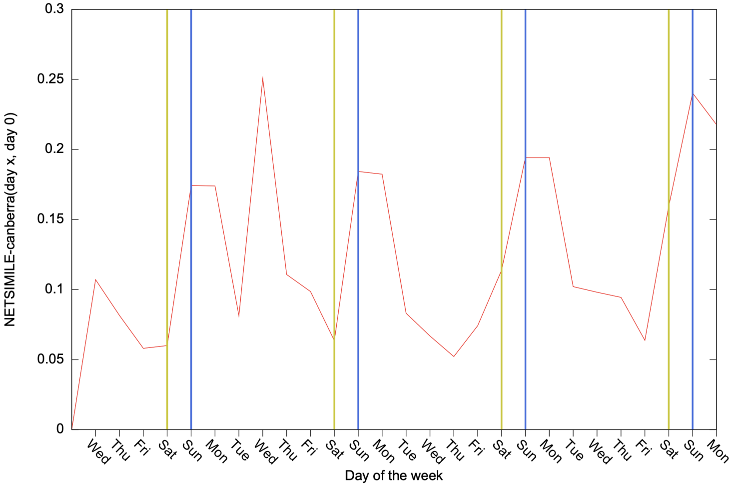
  [Hunter and McIntosh, 1999, Gensler and Sick, 2017]

Netsimile

- an event exists in G_{j+1}, if G_{j+1} is very different than G_j
- for each node calculate 7 local and egonet-based measures
  - degree
  - clustering coefficient
  - average degree of neighbours
  - average clustering coefficient of neighbours
  - number of edges in the egonet
  - number of edges outgoing from the egonet
  - number of neighbours of the egonet
- combine into a signature vector and compare

[Berlingerio et al., 2012]

Netsimile algorithm



(a) NetSimile between each day and day 0 in Yahoo! IM

[Berlingerio et al., 2012]

bursting events

- an influential work by Kleinberg observed that events are characterized by bursting activity

  [Kleinberg, 2003]

  – e.g., people discuss a topic intensively during the short period of time

- recent works rely on this connection

  [Abdelhaq et al., 2013, Kunneman and van den Bosch, 2014]

hierarchical events

- time intervals of the events
- events are not isolated
- they have different importance
  - local and global events can happen simultaneously
  - a large event amy consist of several smaller events
- thus, hierarchical event models are meaningful

  [Dong et al., 2015, Li et al., 2014]

temporal event detection

we want to detect

- additional structural features, e.g.,
  - periodicity [Kunneman and Van den Bosch, 2015]
  - meta-information, e.g., text or tags of messages

most practical event-detection tools

- are application specific
  - breaking news or trends on twitter

    [Batal et al., 2012, Aggarwal and Subbian, 2012]
  - use multiple time sequence analysis techniques as building blocks

    [Rayana and Akoglu, 2016]

spatiotemporal event detection

detailed survey [Shi and Pun-Cheng, 2019]

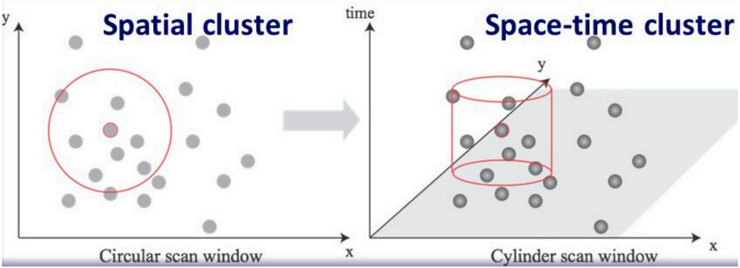
- consider time and the (geo-)location of an event
- sources of spatial data
  - GPS devices / smart phones
  - geo-tagged messages in online social networks
- typical approaches model the data as a set of geo-locations associated with activity measurements
- given a set of locations with activity measures, we want to find a subset of locations that are close to each other and have abnormal activity pattern
- in spatiotemporal setting, one is also interested in finding the time interval (moment) of an event

spatiotemporal event detection: scan statistics

- a classic family of methods is spatial and spatiotemporal scan statistics
- scan over the space and time windows to identify regions of data generated by a non-random process

spatiotemporal event detection: scan statistics

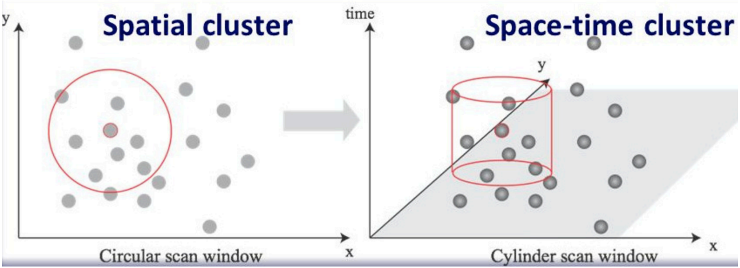
- a seminal paper : spatial scan statistics [Kulldorff, 1997]
- scan a circular spatial window and test the non-randomness of data against Poisson or Bernoulli baseline process



[Takahashi et al., 2004]

spatiotemporal event detection: scan statistics

- later the approach was extended to spatiotemporal scans with cylindric windows
- similar works explore other types of statistics and tests [Neill, 2006, Qian et al., 2014].



[Takahashi et al., 2004]

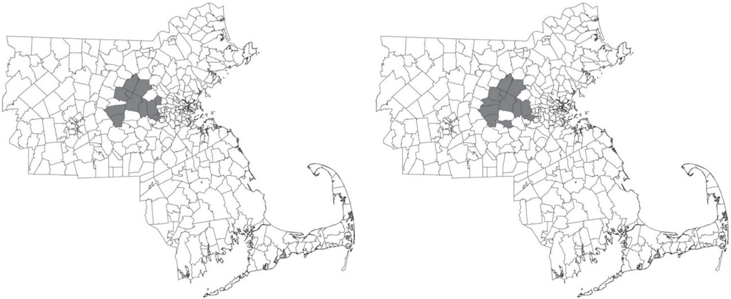
flexible scans

- flexible spatial scan-statistics
- first, divide the entire area into many small regions
  - the location of each region is the administrative population centroid
- next, the set of irregularly shaped windows: concentric circles and connected regions
  - k is a pre-specified maximum length of cluster
- similar idea is used in the flexible space-time scan statistics
- both of these are fitted to a small cluster size

[Takahashi et al., 2008]

flexible scans

simulated disease maps in the Tokyo Metropolitan area



(c) Respiratory (flexible on Aug.12) (d) Respiratory (flexible on Aug.15)

[Takahashi et al., 2008]

structural event

- structural event
  - set of interconnected abnormal nodes
  - no assumptions on geo-desic distances
- e.g., the edge weights represent similarity of nodes
  - similarities between twitter users in preferences, language, frequently visited locations, etc.

- scan extension to graph model [Liu et al., 2016b]
- scan through a graph neighborhood — a set of interconnected nodes
- dense subgraph detection
  - e.g., [Charikar, 2000, Khuller and Saha, 2009]

semantic event detection

- define event as an emerging/bursting/unusual topic in social media, or
- use textual information to supplement and support event detection
  - meaning of the event
  - more robust event detection
- simplest use of textual information monitor the frequencies of separate key words [Lappas et al., 2012]
- efficient for predefined events, vocabulary is known
- more general approach: topic modeling to identify the event vocabulary
- combine with other event-related information
  - e.g., the geo-tags of tweets

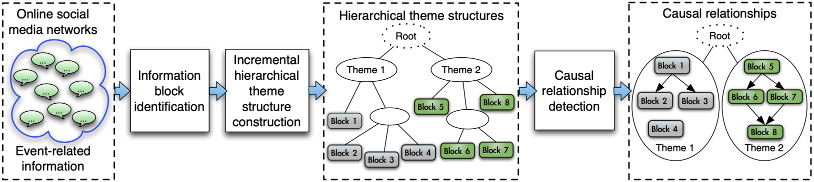
[Hong et al., 2012, Kling et al., 2014]

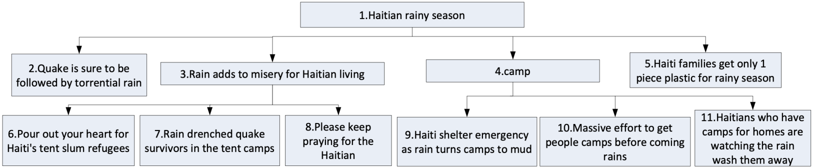
ETree

- aggregate semantically similar (based on *n*-grams) tweets into information blocks
- model an event in twitter as a tree of information hierarchy, where nodes are subtopics
- each subtopic is a directed graph of information blocks, where edges are potential causal relationships
- the causal estimates rely on content similarity and temporal relevance
- assemble a topic tree by greedy heuristic

[Gu et al., 2011]

ETree





[Gu et al., 2011]

finding important nodes

PageRank

- classic approach for measuring node importance
- listed in the top-10 most important data-mining algorithms

  [Wu et al., 2008]

- numerous applications
  - ranking web pages
  - trust and distrust computation
  - finding experts in social networks
  - …

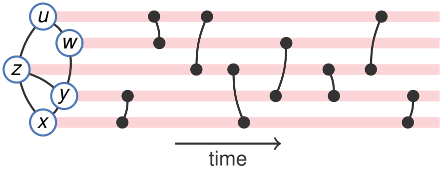
PageRank

- PageRank defined as the stationary distribution of a random walk in the graph
- inherently a static process
- however, many modern networks can be viewed as a sequence (stream) of edges
  - temporal network : $G = (V, E)$, with $E = \{(u, v, t)\}$
  - examples : twitter, instagram, IMs, email, ...
- what is an appropriate PageRank definition for temporal networks?

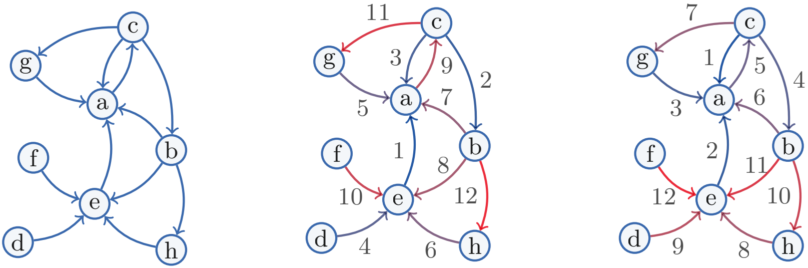
temporal networks

network nodes interact with each other

(e.g., a “like”, a repost, or sending a message to each other)



motivating example



(a) static network

(b) temporal network

(c) temporal network

research questions and objectives

- extend PageRank to incorporate temporal information and network dynamics
- adapt PageRank to reflect changes in network dynamics and node importance
- estimate importance of a node u at any given time t

[Rozenshtein and Gionis, 2016]

dynamic PageRank vs. temporal PageRank

- extensive work on dynamic PageRank
- dynamic PageRank computation :

  maintain correct PageRank during network updates
  - e.g., edge additions / deletions
- computation should return the static PageRank at a given network snapshot
- for edges present in a snapshot, order does not matter

[Rozenshtein and Gionis, 2016]

static PageRank

- graph $G = (V, E)$
- corresponding row-stochastic matrix $P \in \mathbb{R}^{n \times n}$
- personalization vector $\mathbf{h} \in \mathbb{R}^n$
- PageRank is the stationary distribution of a random walk, with restart probability $(1 - \alpha)$

$$\pi(u) = \sum_{v \in V} \sum_{k=0}^{\infty} (1 - \alpha)\alpha^k \sum_{\substack{z \in \mathcal{Z}(v,u) \\ |z|=k}} h(v) \Pr[z \mid v]$$

where, $\mathcal{Z}(v, u)$ is the set of all paths from v to u

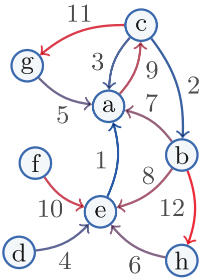
and $\Pr[z \mid v] = \prod_{(i,j) \in z} P(i, j)$

temporal PageRank

- make a random walk only on temporal paths

  e.g., time-respecting paths

  time-stamps increase along the path



$c \rightarrow b \rightarrow a \rightarrow c$: time respecting

$a \rightarrow c \rightarrow b \rightarrow a$: not time respecting

temporal PageRank

- intuition : probability of visiting node u at time t
  given a random walk on temporal paths
- need to model probability of following next temporal edge
  - we use an exponential distribution
- temporal PageRank definition

$$r(u,t) = \sum_{v \in V} \sum_{k=0}^{t} (1-\alpha)\alpha^k \sum_{\substack{z \in \mathcal{Z}^T(v,u|t) \\ |z|=k}} \Pr'[z|\, t]$$

$\mathcal{Z}^T(v, u \mid t)$ set of temporal paths from v to u until time t

computation

- simple online algorithm
- $r(u, t)$: temporal PageRank estimate of u at time t
- $s(u, t)$: count of active walks visiting u at time t

input : E, transition probability β, jumping probability α

$\boldsymbol{r} = \mathbf{0}$, $\boldsymbol{s} = \mathbf{0}$;

foreach $(u, v, t) \in E$ **do**

  $\boldsymbol{r}(u) = \boldsymbol{r}(u) + (1 - \alpha)$;

  $\boldsymbol{r}(v) = \boldsymbol{r}(v) + (\boldsymbol{s}(u) + (1 - \alpha))\alpha$;

  $\boldsymbol{s}(v) = \boldsymbol{s}(v) + (\boldsymbol{s}(u) + (1 - \alpha))(1 - \beta)\alpha$;

  $\boldsymbol{s}(u) = (\boldsymbol{s}(u) + (1 - \alpha))\beta$;

normalize $\boldsymbol{r}$;

return $\boldsymbol{r}$;

static vs. temporal PageRank

- temporal PageRank is designed to capture changes in network dynamics and concept drifts
- what if the edge distribution is stable?

static vs. temporal PageRank

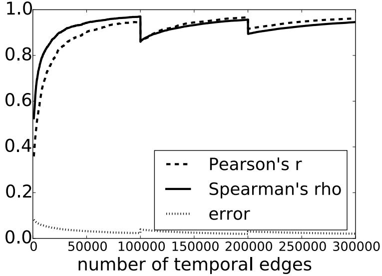
- consider static network $G_S = (V, E_S, w)$
- time period $[1, \ldots, T]$
- construct temporal network $G = (V, E)$ by sampling edges proportionally to their weight

proposition :

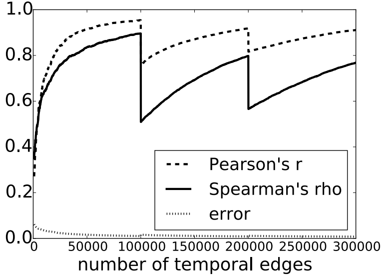
as $T \to \infty$, the temporal PageRank on G
converges to the static PageRank on G_S,
with personalization vector equal to weighted out-degree

[Rozenshtein and Gionis, 2016]

experiment — adaptation to concept drift



(a) *Facebook*



(b) *Twitter*

[Rozenshtein and Gionis, 2016]

diffusion analysis and influence spreading

diffusion analysis and influence spreading

- propagation models
  - used to study disease spreading or information cascade in the network
- activity spreading: virus, information, idea, rumor
- applications: epidemiology, information security, marketing
- why use models?
  - facilitate mathematical analysis of propagation processes
  - have intuitive interpretation
  - proven to be realistic by empirical studies
- extensive survey in the book [Shakarian et al., 2015]

standard models

- susceptible-infected (SI) model
  - SIR, SIRS, other variants
- independent cascade (IC) model
- iinear threshold (LT) model
- shortest path (SP) model

static models: assumptions

- all models have similar implicit assumptions on temporality:

1. uniform time steps
2. interactions happen at each time step and are independent

drawbacks of static models

- large heterogeneity in the time instances of real interactions

  [Barabasi, 2005, Candia et al., 2008, Leskovec and Horvitz, 2008]

- burstiness in communication patterns
- periodic activity changes
- causal relationships between interactions

temporal propagation models

- intuitive extensions from static graphs to temporal graphs
- add distributions (e.g., Poisson or power-law) of the intervals between interactions (latencies)

  [Vazquez et al., 2007, Min et al., 2011]
- realistic generalizations of well-studied models

  [Karsai et al., 2011, Candia et al., 2008]
- continuous time, partially observed graph
- develop mathematical analysis for novel and generalized models

  [Harris, 2002, Fernández-Gracia et al., 2011]

typical problem formulations

- immunization strategies
- influence maximization
- seed and cascade reconstruction

static immunization strategies

- main aspects differentiating the research works:
  - assumptions about the spreading model
  - assumptions about the network structure
  - whether the whole network is observable
- both assumptions on the network structure and on the infection propagation are crucial
- results may not hold for any general network and real infection

  [Newman, 2003, Pastor-Satorras and Vespignani, 2002a].

static immunization strategies

- simple model-blind strategies, such as random immunization, perform moderately well in different scenarios

  [Pastor-Satorras and Vespignani, 2002b, Madar et al., 2004]

- better results on real-world networks can be achieved by immunizing nodes with high connectivity

  [Pastor-Satorras and Vespignani, 2002b, Dezső and Barabási, 2002].

- requires explicit knowledge of the network structure and it is impractical for real applications

static immunization strategies

- [Cohen et al., 2003] overcomes this drawback by employing acquaintance immunization strategy:
- immunization of random neighbors of randomly selected nodes leads to immunization of the most central nodes without knowing any global information about the network

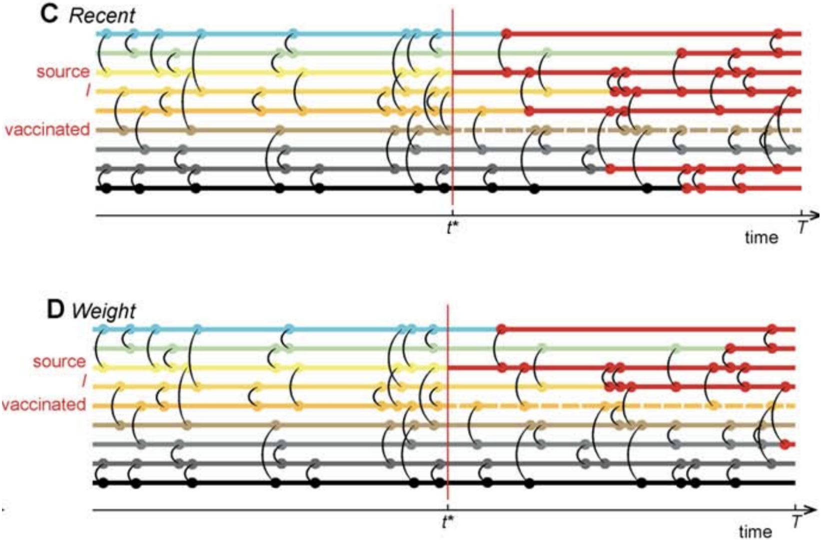
temporal immunization strategies

- adjust successful static strategies
- e.g., Cohen's neighborhood vaccination scheme

  [Lee et al., 2012]

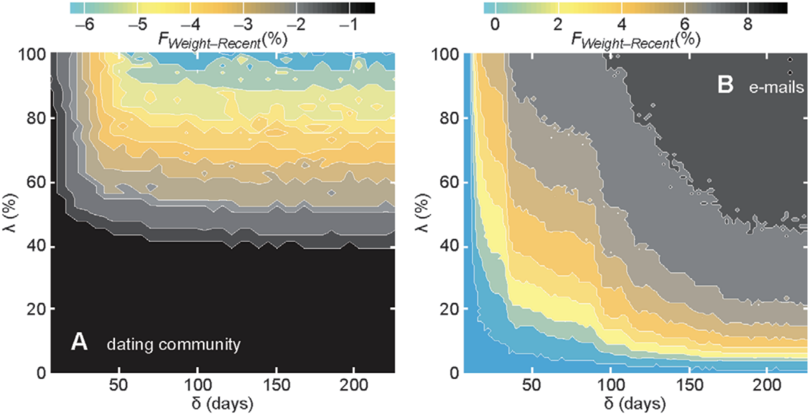
- two vaccination strategies
- recent :
  - ask a random individual i to name its most recent contact and vaccinate this person
- weight :
  - ask a random individual i to name its most frequent contact since some time t

2 protocols



[Lee et al., 2012]

temporal immunization strategies



recent is the most efficient method for the most of the datasets

[Lee et al., 2012]

temporal immunization strategies

- full knowledge of the temporal graph:
  - vaccinations of nodes with high temporal degree, temporal betweenness, or other type of centrality

    [Yu et al., 2010, Starnini et al., 2013, Génois et al., 2015]

- another line of works
- find persistent communication patterns to approximate the communication structure in future
  - apply standard vaccination on the predicted graph

    [Valdano et al., 2015, Gauvin et al., 2015]

    [Mantzaris and Higham, 2016]

static influence maximization

- how to select theinitial set of infected nodes (seeds), such that the speed, size, or other spread characteristics are optimized
- applications in marketing and network design
- influence maximization problem was introduced by [Kempe et al., 2003] in the IC and LT models
- find a set of k seed nodes, such that the expected number of nodes activated by the infection cascade is maximized

static influence maximization

- NP-hard [Kempe et al., 2003]
- simple greedy algorithm with approximation guarantee
- influence maximization problem was been studied for many different variants of other models, constraints, and objective functions
- many practical heuristics and approximations

  [Chen et al., 2009, Chen et al., 2010, Tang et al., 2014]

temporal influence maximization

- intuitive approach to reflect temporality:
  - sequence of graphs (or snapshots)
  - each time step of propagation corresponds to propagation over the corresponding graph
  - all interactions within one time step happen simultaneously
- related papers:

  [Aggarwal et al., 2012, Zhuang et al., 2013, Gayraud et al., 2015]

temporal influence maximization

- another approach:
- incorporate time into the diffusion model as distribution of intervals between the interactions
- different types of models and interval distributions

  [Chen et al., 2012, Liu et al., 2012, Rodriguez and Schölkopf, 2012, Du et al., 2013]
- the most realistic approachable setting?
- the latest promising research:
  - infer propagation model parameters from the data

    [Rodriguez et al., 2011, Gomez-Rodriguez et al., 2016]

influence maximization in the continuous model

- use fully continuous time model of diffusion

  [Rodriguez et al., 2011]
- pairwise transmission likelihood:
- define $f(t_j \mid t_i; \alpha_{i,j})$ as the conditional likelihood of transmission between a node i and a node j,
  - t_i and t_j are infection times and
  - $\alpha_{i,j}$ is the transmission rate
- assume that the likelihood depends on:
  - the pairwise transmission rate $\alpha_{i,j}$ and
  - the time difference $(t_j - t_i)$
- consider the exponential distribution of model pairwise interactions

[Gomez-Rodriguez et al., 2016]

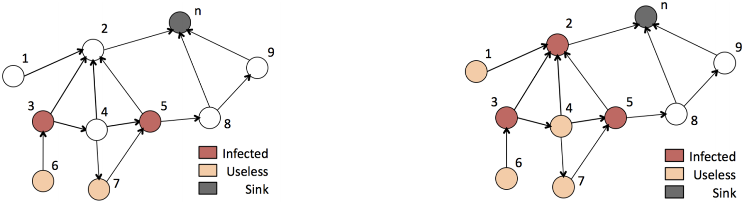
influence maximization in the continuous model

- given a diffusion process that started in the set of source nodes A
- $N(A; T)$ is the number of nodes infected up to time T
- the influence function $\sigma(A; T)$ as the average total number of nodes infected up to time T, i.e., $\sigma(A; T) = \mathbb{E}N(A; T)$.
- continuous time influence maximization problem:
  - find the set of source nodes A in a diffusion network G that maximizes the influence function $\sigma(A; T)$
- i.e., $A = \arg\max_{|A| \leq k} \sigma(A; T)$

[Gomez-Rodriguez et al., 2016]

influence maximization in the continuous model

- efficient pruning based on identifying ‘blocked’ nodes
- infection time of a node is the length of the stochastic shortest path



(a) t_1: $|I| = 2$, $|U_n| = 2$, $|X_n| = 4$

(b) t_2: $|I| = 3$, $|U_n| = 4$, $|X_n| = 7$

[Gomez-Rodriguez et al., 2016]

seed and cascade reconstruction

- given someobserved data about the infection
  - e.g., a small subset of infected nodes,

  the goal is to find the most probable seed nodes

- other versions:
  - find the most probable cascades
- the order of infection (who got infected from whom)
- these works are data-driven:
  - it is essential that the assumed propagation model matches the actual infection flow in the network

seed and cascade reconstruction

- applications:
  - epidemiology (who was the patient zero?)
  - influencer discovery (who was the source of information?)
- a number of different approaches
  - find a single source under the SI model [Shah and Zaman, 2011]
  - multiple seeds [Prakash et al., 2012]
  - *k* seeds under the IC model [Lappas et al., 2010]
- the most recent papers
  - take advantage of the recorded infection order [Sefer and Kingsford, 2016].

temporal reconstruction

- the problems formulated in this setting tend to be either
  - oversimplified versions of static reconstruction or
  - become too hard or ill-posed
- knowing the history of interactions allow to reconstruct feasible paths of infection and prune unfeasible
- any noise or missing information adds uncertainty
- need more assumptions about the noise and information available

temporal reconstruction

- some problem formulations :
- reconstruct the cascade given the sequence of graph snapshots along with node-status information

  [Feizi et al., 2016, Sefer and Kingsford, 2016]
- reconstruct an SI cascade from one sampled snapshot with all information

  [Sundareisan et al., 2015]
- while there are methods to handle partially observed cascade for static graphs, in temporal graphs most of works rely on noise-free data
- the knowledge of the diffusion model in crucial
- see survey paper: [Holme, 2015]

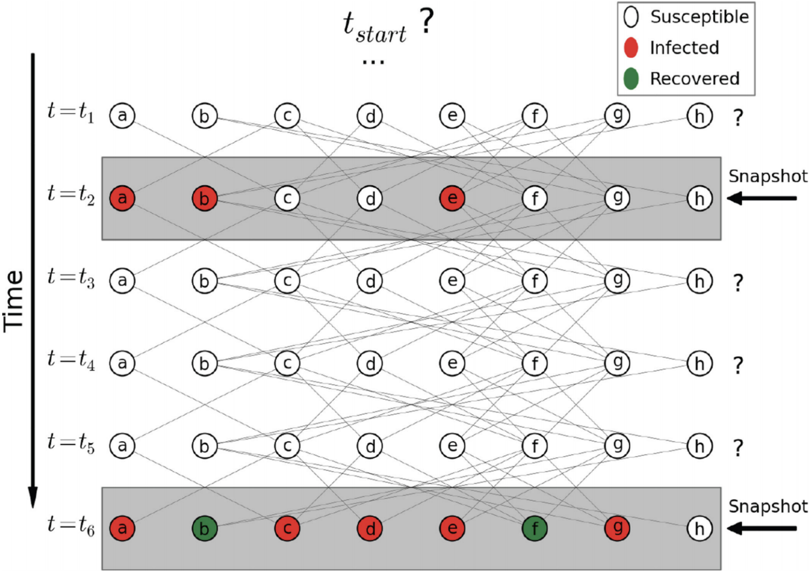
history reconstruction

[Sefer and Kingsford, 2016]

- SEIRS diffusion dynamics over directed graph $G = (V, E)$
- SEIRS states are Susceptible (S), Exposed but not contagious (E), Infected and contagious (I), and previously infected but Recovered (or immune to the infection) (R)

- given: a graph $G = (V, E)$, state transition probabilities $(p_{u,v}, e2i_v, i2s_v, i2r_v, r2s_v)$, and a collection of diffusion snapshots = $\{D_t\}$, with $D_t \in T_D$
- each snapshot records the state of every node at a single time point, partitioning them into $V = S_t \cup E_t \cup I_t \cup R_t$
- the goal is to infer the past states (susceptible, exposed, infected and recovered) of every node at every time

history reconstruction



[Sefer and Kingsford, 2016]

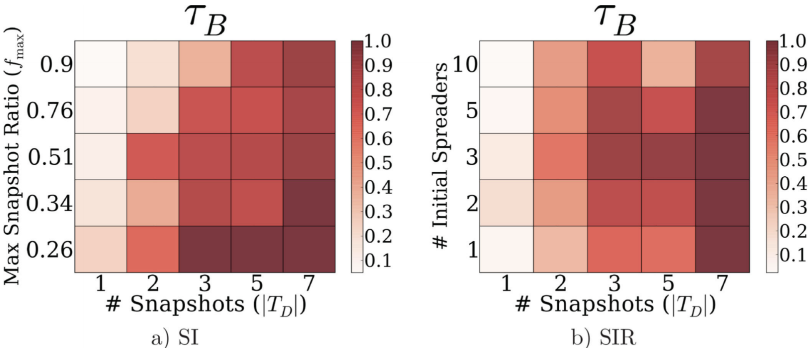
history reconstruction

[Sefer and Kingsford, 2016]

- proposed solution :
  - maximum likelihood history given diffusion snapshots that may come from multiple time points
- algorithm called DHR-sub (submodular history reconstruction on discrete dynamics)
- reconstructs the history before the earliest measurement:
  - greedily maximize the non-monotone submodular log-likelihood at each previous time step
- reconstructs the history between the consecutive diffusion data time points:
  - non-monotone submodular maximization under matroid base constraints
- speedups and approximations

[Sefer and Kingsford, 2016]

history reconstruction



[Sefer and Kingsford, 2016]

network summarization

network summarization

- aims to simplify and explain the high-level structure of complex real graphs
- many different problem formulations and techniques:
  - recent survey [Liu et al., 2016a]

motivation and applications

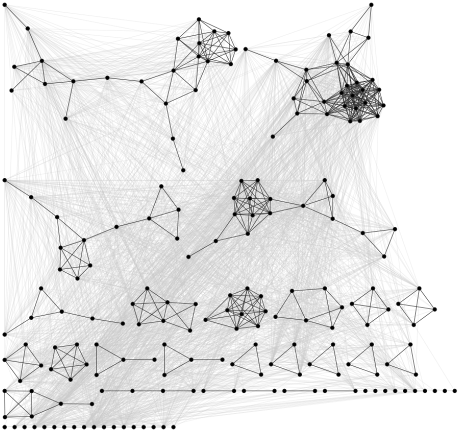
- fast and interactive large-graph analysis:
  - summaries decrease space and memory required for the storage and processing of real-world networks
- clear human understandable visualization
- noise elimination: filter out insignificant structural fluctuations in networks and preserve only prominent patterns

approaches to summarization

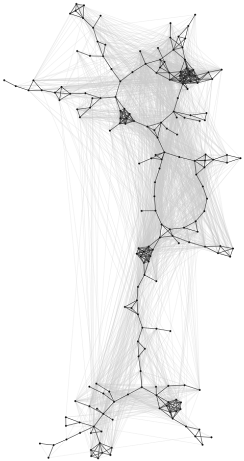
- sparsification
- aggregation / compression
- non-graph summary

sparsification

- remove somewhat unimportant edges or/and nodes
- preserving certain local or/and global structures
- important properties to preserve are cuts, community structures, distances, spectral properties, etc.



(a) Quadrilateral Simmelian Backbone



(b) Quadrilateral Simmelian Backbone with UMST

[Hamann et al., 2016]

sparsification

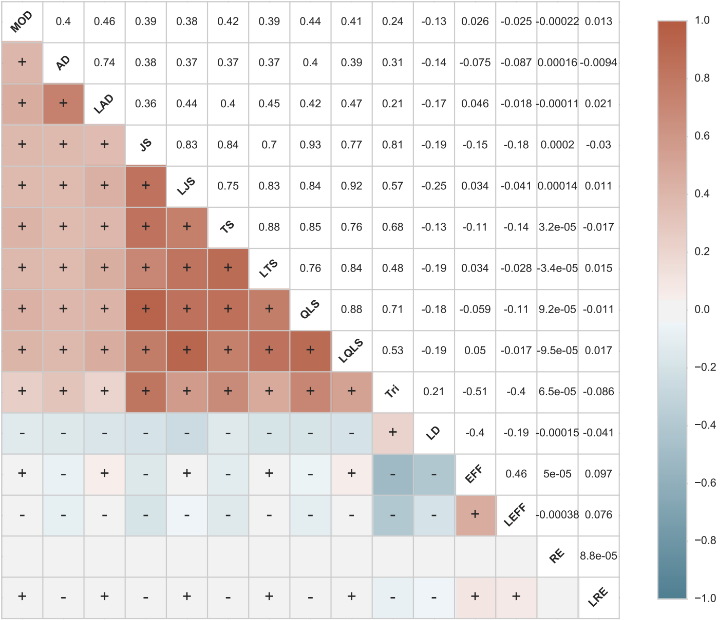
- sparsification problems are often formulated as optimization problems:
  - minimize some kind of graph approximation (reconstruction) error
  - while sparsifying as much as possible
- examples:
  - preservation of distances between nodes and connectivity [Elkin and Peleg, 2005, Zhou et al., 2010]
  - cuts [Ahn et al., 2012]
  - spectral graph properties [Batson et al., 2013]
  - various types of social network-specific characteristics
- survey: [Hamann et al., 2016]

comparison

- random edge (RE)
- triangle counts (Tri)
- Jaccard similarity (JS) [Satuluri et al., 2011]
- simmelian backbones (TS, QLS) [Nick et al., 2013]
- edge forest fire (EFF) [Leskovec and Faloutsos, 2006]
- algebraic distance (AD) [Chen and Safro, 2011]
- local degree (LD) [Hamann et al., 2016]
- "local" versions of all mentioned methods [Hamann et al., 2016]

[Hamann et al., 2016]

comparison


MOD 0.4 0.46 0.39 0.38 0.42 0.39 0.44 0.41 0.24 -0.13 0.026 -0.025 -0.00022 0.013
+ AD 0.74 0.38 0.37 0.37 0.37 0.4 0.39 0.31 -0.14 -0.075 -0.087 0.00016 -0.0094
+ + LAD 0.36 0.44 0.4 0.45 0.42 0.47 0.21 -0.17 0.046 -0.018 -0.00011 0.021
+ + + JS 0.83 0.84 0.7 0.93 0.77 0.81 -0.19 -0.15 -0.18 0.0002 -0.03
+ + + + LJS 0.75 0.83 0.84 0.92 0.57 -0.25 0.034 -0.041 0.00014 0.011
+ + + + + TS 0.88 0.85 0.76 0.68 -0.13 -0.11 -0.14 3.2e-05 -0.017
+ + + + + + LTS 0.76 0.84 0.48 -0.19 0.034 -0.028 -3.4e-05 0.015
+ + + + + + + QLS 0.88 0.71 -0.18 -0.059 -0.11 9.2e-05 -0.011
+ + + + + + + + LQLS 0.53 -0.19 0.05 -0.017 -9.5e-05 0.017
+ + + + + + + + + Tri 0.21 -0.51 -0.4 6.5e-05 -0.086
- - - - - - - - - + LD -0.4 -0.19 -0.00015 -0.041
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- - - - - - - - - - - + LEFF -0.00038 0.076
RE 8.8e-05
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1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0

comparison

- random edge deletion:
  - performs surprisingly well
  - retains a wide range of properties
- simmelian backbones, Jaccard similarity and algebraic distance:
  - prefer intra-cluster edges
  - do not keep global structures
- local degree:
  - preserves shortest paths
  - overall connectivity of the network
- forest fire sampling edge scoring:
  - depends strongly on the specific network's structure
  - good at preserving connectivity

[Hamann et al., 2016]

aggregation / compression

- super graph:
  - nodes are grouped into supernodes and
  - edges between the super nodes form superedges
- graph aggregation can be formulated as an optimization problem
  - minimizing reconstruction error
  - preserve some properties
- the preserved properties are similar to sparsification problems

aggregation / compression

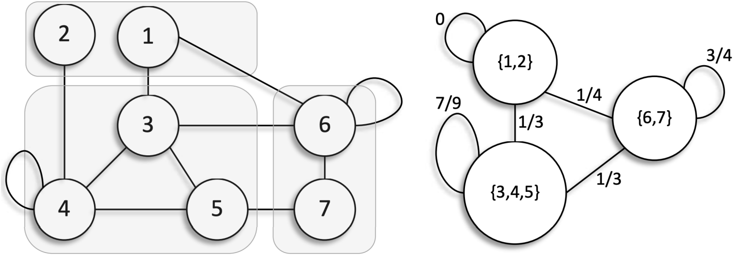
- some examples:
  - node aggregation to approximatenode degree and eigenvector centrality

    [LeFevre and Terzi, 2010, Riondato et al., 2017]
  - edge aggregation to preserve the weights of superedges or strengths of the paths

    [Toivonen et al., 2011]
- common heuristic is to build a supergraph based on clustering

  [Abello et al., 2006, Clémençon et al., 2012]

compression example

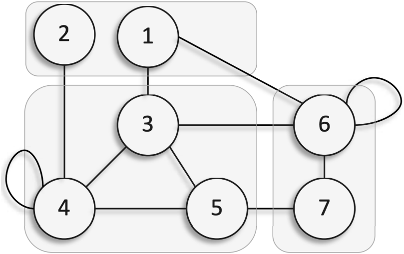


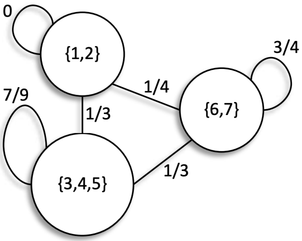
- graph $G = (V, E)$
- number k
- A_G : adjacency matrix of G

- k-summary S of G is a complete undirected weighted graph $S = (V', V' \times V')$
- where V' is a disjoint k-partition of V

[Riondato et al., 2017]

compression example





- the vertices of S are called supernodes, edges are superedges
- each superedge e_{ij} has a weight, corresponding to the density of edges between V_i and V_j:

$$d_G(i,j) = \frac{\sum_{i' \in V_i, j' \in V_j} A_G(i',j')}{|V_i||V_j|}$$

[Riondato et al., 2017]

- density matrix of S as the $k \times k$ matrix A_S with entries $A_S(i,j) = d_G(i,j)$, $1 \leq i,j \leq k$
- $A_S \in \mathbb{R}^k \times k$ can be lifted to the matrix $A^{\uparrow} \in S \in \mathbb{R}^n \times n$ as $A_S^{\uparrow}(v,w) = A_S(s(v), s(w))$
- summarization problem: find the k-summary to minimize the error $err(A_G, A_S^{\uparrow}) = ||A_G - A_S^{\uparrow}||_p$

	1	2	3	4	5	6	7
1	0	0	1/3	1/3	1/3	1/4	1/4
2	0	0	1/3	1/3	1/3	1/4	1/4
3	1/3	1/3	7/9	7/9	7/9	1/3	1/3
4	1/3	1/3	7/9	7/9	7/9	1/3	1/3
5	1/3	1/3	7/9	7/9	7/9	1/3	1/3
6	1/4	1/4	1/3	1/3	1/3	3/4	3/4
7	1/4	1/4	1/3	1/3	1/3	3/4	3/4

[Riondato et al., 2017]

non-graph summary

- represent some interesting, characterizing, or otherwise important structures observed in the graph
  - e.g. a set of tightly interconnected nodes (communities)
  - graph can be summarized as a set of communities, ignoring other parts [Lancichinetti et al., 2011, Perozzi and Akoglu, 2018]

non-graph summary

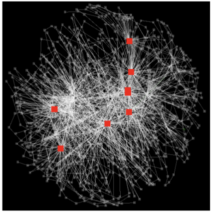
- other examples:
  - motif counting
    (counting small subgraphs of restricted size)
    [Itzhack et al., 2007]
  - finding frequent subgraphs
    [Jiang et al., 2013]

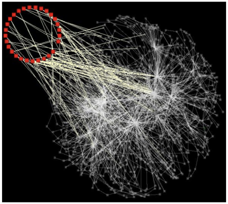
- other approaches develop specialized vocabulary to encode a large graph.
- e.g., summarize by a set of chains, stars, cliques, and bipartite cores
  [Koutra et al., 2015]
- this framework can be further extended to domain-specific vocabulary constructed by an expert

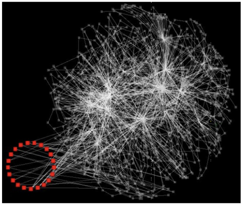
vocabulary-based summarization

- vocabulary: full and near cliques (fc, nc), full and near bipartite cores (fb, nb), stars (st), and chains (ch)
- encode the graph using MDL-base encoding:

  graph = vocabulary + noise







- more approaches in the survey [Liu et al., 2016a]

[Koutra et al., 2015]

temporal graph summarization

- time-related changes are important:
  - summarized patterns and substructures may not be persistent in time
  - the elements of a pattern can be frequent in different distant time periods and not frequent in a continuous time interval
- purely temporal patterns may occur:
  - substructures may change in time according to hidden rules
  - e.g., nodes with certain labels may gain centrality over time, while the importance of some other labels may decline

adaptation of existing techniques

- frequent subgraph mining: find persistent graph patterns over a collection of snapshots
- do not take into account how the instances of the same subgraph are located in time

- sequential pattern mining: search for time-ordered patterns in the sequence of snapshots
- network evolutionary patterns [Berlingerio et al., 2009, Wackersreuther et al., 2010]
- ignores structural patters

- time-series analysis: gather node- and structure-dependent statistics over time
- apply segmentation techniques [Ye and Keogh, 2009]
- does not consider network structure

temporal techniques

- summarization of both structural and temporal aspects

- how to define a summary?
- many possible options:
  - a summary can be a short temporal sequence of small graphs,
  - a concise presentation of evolutionary patterns,
  - a representative collection of temporally and topologically frequent patterns

- one common approach to summary definition:
- summary should consist of
  - small structurally “interesting” subgraphs
  - with non-trivial temporal behavior

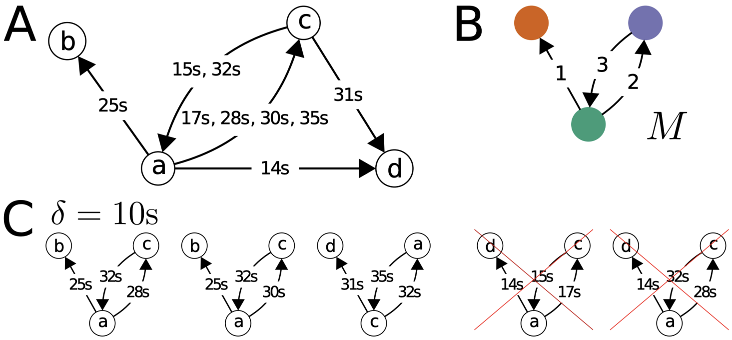
frequent and persistent temporal subgraphs

- definition of temporal subgraphs?
  - undirected or directed subgraphs aggregated with or without frequency edge-weight over short intervals
  - directed acyclic graphs, as they model information flow in the graph
- temporal order of interactions:
  - fixed or flexible
- temporal constrains:
  - window length and/or delays between two interactions
- how to measure counts, frequencies, and importance of the subgraphs?
- how to treat the temporal duplicates of the same edges?
- how to weight patterns by the time span and recency?

temporal motifs

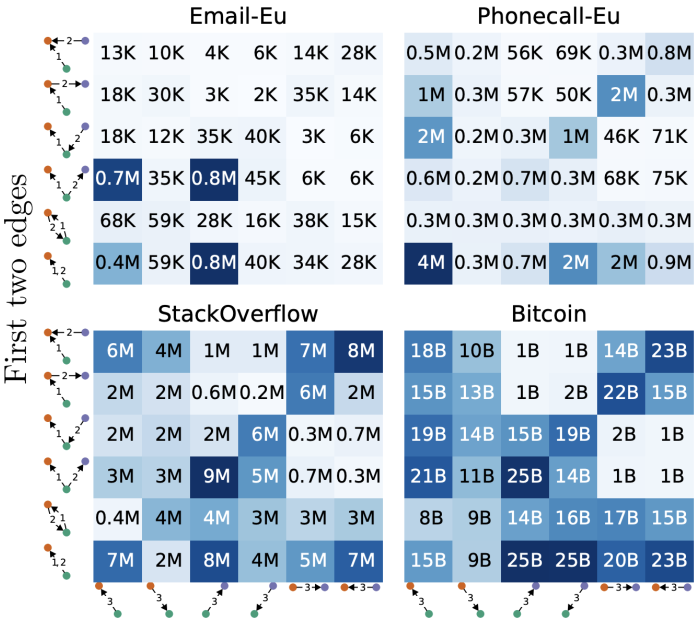
- temporal motif counting
  [Paranjape et al., 2017, Kovanen et al., 2013]:
  - temporal motif is a small subgraph with temporally ordered edges (and/or interval or delay constraints)
- some other works explore temporal graphlets
  - time constrained causal subgraphs
    [Hulovatyy et al., 2015]
    and cyclic patterns
    [Lahiri and Berger-Wolf, 2008]

temporal motifs



δ-temporal motif: a sequence of directed temporally ordered edges which appear within a time window δ

[Paranjape et al., 2017]



[Paranjape et al., 2017]

vocabulary-based summarization

- summarize a temporal graph as a set:
  - subgraphs of a special “most non-random” shape (stars, cliques, bipartite cores, chains), and
  - behavioural temporal patterns (flickering, periodic, oneshot, ranged, and constant patterns)
- use MDL principle to encode whole temporal network by the vocabulary plus noise

[Shah et al., 2015]

larger structures

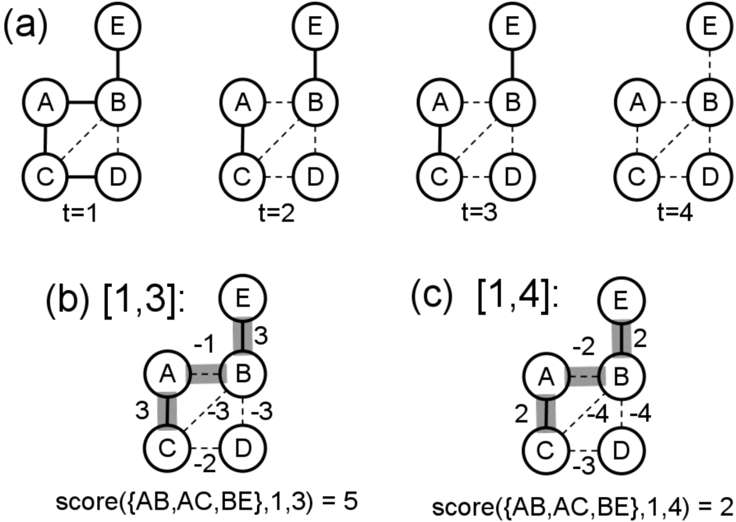
- use larger structures to summarize the network:
  - communities
  - spanning graphs
  - backbones
  - cores
- common approach:
  - given a sequence of graphs
    (snapshot, or sliding-window aggregation)
  - search for communities that are coherent and/or persistent in time
- different measures of community quality and temporal smoothness are used
  [Pietilänen and Diot, 2012, He and Chen, 2015]
- the resulting summary is a trade-off between structural quality and historical consistency

temporal backbones

- $G = (G_1, \ldots, G_F)$ time history $[1, F]$
- $G_i = (V, E)$ have weighted edges $w_i : E \to \mathbb{R}$
- the heaviest temporal subgraph:
- find an interval $[i, j] \subseteq [1, F]$ and a subgraph $G' = (E', V') \subseteq G$, that maximizes

  $$score(G', i, j) = \sum_{e \in E'} \sum_{k=i}^{j} w_i(e)$$
- NP-hard problem
- scalable heuristics

[Bogdanov et al., 2011]



[Bogdanov et al., 2011]

influence-based summarization

- summarizes the flow of information propagation:
  - find influential nodes and information-forwarding connections
- OSNet [Qu et al., 2014]:
  - processes a temporal network in a streaming fashion
  - outputs the subgraphs of influential nodes
  - node importance is calculated based on temporal spreading trees
- [Lin et al., 2008] identify influential nodes and interactions in temporal multi-view social networks
  - networks with edges between different types of entities, e.g., users, photos, and comments
  - explain the evolution of topics over time

agenda

Part I : introduction and motivation

Part II : models of temporal networks

Part III : group work

Part IV : algorithmic frameworks

Part V : data mining problems

Part VI : future challenges

Part VII : group work

part VI

future challenges

temporal community detection: challenges

- large number of problem formulations and variants
- lack fundamental theoretical treatment
  - most of the approaches are heuristics
  - many are combinations of several ideas and algorithms
  - require many parameters and attention to implementation details
- hard to compare methods and choose one for an application
  - few datasets with ground-truth temporal communities
  - synthetic generators are built on various assumptions
  - no standards and benchmarks
- a large number of quality metrics to calculate and compare
- may be misleading if a method is not designed for that particular community definition

temporal community detection: directions

- more systematic approaches, quality guarantees
- interpretability of the results
- visualization
- applications and application-tailored algorithms, e.g., for
  - computational social science
  - temporal network summarization

event detection: challenges

- actively evolving area, application- and data-oriented
- families of problems and methods are considered only for the specific sources of data
  - e.g., a large body of research is focused on the analysis of Twitter data [Atefeh and Khreich, 2015]
- no unified classification for problem settings, research questions, and data requirements
  - recent classifications are based on various aspects:
  - event definitions, online or retrospective detection, specified or unspecified event detection, etc. [Cordeiro and Gama, 2016, Goswami and Kumar, 2016]

event detection: directions

- speed and quality:
- online streaming event-detection techniques are demanded for nearly real-time event detection
- quality: both false events and missed events may have a high price
- more use of multi-modal data:
- text: complex semantic and sentiment analysis is rare
- high-resolution interaction patterns: “who talked to whom about what and what happened then” are also often not considered

diffusion analysis: challenges

- influence maximization:
  - what is the most realistic approachable setting?
  - the latest promising research focuses on inferring the parameters of a propagation model from the data, including latency distributions [Rodriguez et al., 2011, Gomez-Rodriguez et al., 2016]
- reconstruction:
  - received little attention
  - the problems formulated in this setting tend to be either oversimplified versions of static reconstruction or become too hard or ill-posed
  - most of the works rely on noise-free data
  - the assumption of diffusion model is crucial

diffusion analysis: open directions

- models:
  - temporal diffusion models are proposed, but the theoretical properties of many of them are not yet well studied
  - the applications and limitations are not yet well understood
- immunization strategies:
  - not extensively studied yet
  - most of the approaches are based on heuristics

summarization challenges

- meaningful summary vocabulary
- diversity of summarizing substructures is vast

  [Perozzi and Akoglu, 2018, Koutra et al., 2015, Jiang et al., 2013])
- which summaries are preferable and in which applications?
- summaries useful for a general network exploration by a non-expert analyst?

summarization challenges

- fast and light-weighted algorithms
- interactive analysis
- have a hierarchical structure, which is possible to browse
  - similar to a visual analytic tool OntoVis, which constructs some type of graphical summaries [Shen et al., 2006]
- multi-level summarizations:
- use all available attributes in the temporal networks
  - text, geotags, propagation patterns...

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